



Numerical Simulation of Lid-Driven Cavity Flow by the Lattice Kinetic Scheme

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Abstract

A new numerical method for incompressible Newtonian fluid flows based on the lattice Boltzmann method (LBM) is proposed. In this method, the relaxation time in the BGK collision term is kept at unity taking account of numerical stability. Compared with the lattice Boltzmann method, the lattice kinetic scheme (LKS) can save the computer memory since there is no need to store the density distributions. The implementation of the boundary condition is direct and just the same as the convectional Navier-Stokes solvers. In this study, the lattice kinetic scheme is applied to two dimensional lid-driven cavity flow as a benchmark problem in fluid dynamics. Obtained results have been compared very well with single relaxation time (SRT) results and also available data in the literature. The simulations indicate that the method can be used for different practical fluid flows very well.

Keywords: lattice kinetic, lattice Boltzmann method, lid-driven cavity, single relaxation time.

Introduction

In recent years, the lattice Boltzmann method has been developed into an alternative promising tool for fluid mechanics. It has been widely used in many kinds of complex flows such as turbulent flows, multiphase flows, and microflows [1]. However, there are still some items in need of further study. One is the collision model. The Bhatnagar-Gross-Krook (BGK) model with a single relaxation time is usually used for the collision term. The shortcomings of the BGK model are pointed out in the works of d'Humières [2] and Lallemand and Luo [3]. The other is the boundary condition. The bounce-back scheme in the LBM was originally taken from the LGA method. Although this heuristic scheme is very simple to implement, it is found to be the first order in the numerical accuracy at the boundaries [4,5]. In order to improve the numerical accuracy, other boundary treatments have been proposed. It appears, however, that the extension of these treatments to the complex boundary surface is difficult. Chen et al. [6] proposed a boundary condition using a second-order extrapolation scheme of the distributions in the flow to obtain the unknown particle distribution functions on the boundaries. When the flow problems with complex geometries, especially in the three dimensions are encountered, the determination of the unknown particle directions is troublesome. All the implementations are not so direct since on the boundaries the macroscopic variables, not the density distributions, are given.

Related to these two difficulties, a lattice kinetic scheme for the incompressible viscous flows was developed by Inamuro [7]. This scheme is based on the idea that if the dimensionless relaxation time in the LBM with the BGK model is set to unity, the macroscopic variables such as velocity components and density instead of the density distribution functions become the dependent variables in the computation so that the numerical stability can be obtained for relatively high Reynolds number flows; nevertheless one is able to determine the fluid viscosity using a constant parameter appearing in the additional term of the equilibrium distribution function. As compared to the standard LBM, this scheme can save computer memory because there is no need to store the density distribution functions. The implementation of the boundary condition is very easy since on the boundaries only the macroscopic variables rather than the density distributions are needed as for the conventional Navier-Stokes solvers. This feature is very useful when the flow problems with complex geometry are concerned.

Peng et al. [8] developed this idea for simulation of fluid flows on arbitrary meshes using technique of Taylor series expansion and least-square-based lattice Boltzmann method. Recently, Yoshino et al. [9] have employed this method to simulate incompressible non-Newtonian fluid flows.

In this paper, a lattice kinetic scheme for incompressible viscous flows is used to simulate two dimensional lid-driven cavity flows in laminar region. To verify the accuracy of the scheme, obtained results have been compared with available standard results.

Theory

Before introducing lattice kinetic scheme, we will give a brief description about the original lattice Boltzmann method.

A. Lattice Boltzmann method

The evolution equation for the density distribution $f_\alpha(x, t)$ in the two dimensions with the particle velocity e_α can be written as

$$f_\alpha(x + e_\alpha \Delta t, t + \Delta t) = f_\alpha(x, t) - \frac{[f_\alpha(x, t) - f_\alpha^{eq}(x, t)]}{\tau} \quad (1)$$

where t is the single relaxation time, f_α^{eq} is the corresponding equilibrium density distribution function,