1.1. Fluid Element Kinematics

1. The three components of velocity in a flow field are given by

$u = x^2 + y^2 + z^2$

$v = xy + yz + z^2$

$w = -3xz - z^2/2 + 4$

(a) Determine the volumetric dilatation rate and interpret the results.

(b) Determine an expression for the rotation vector. Is this an irrotational flow field?

2. A viscous fluid is contained in the space between concentric cylinders. The inner wall is fixed, and the outer wall rotates with an angular velocity (See Fig. 1a). Assume that the velocity distribution in the gap is linear as illustrated in Fig. 1b. For the small rectangular element shown in Fig. 1b, determine the rate of change of the right angle due to the fluid motion. Express your answer in terms of r_0, r_i, and ω.

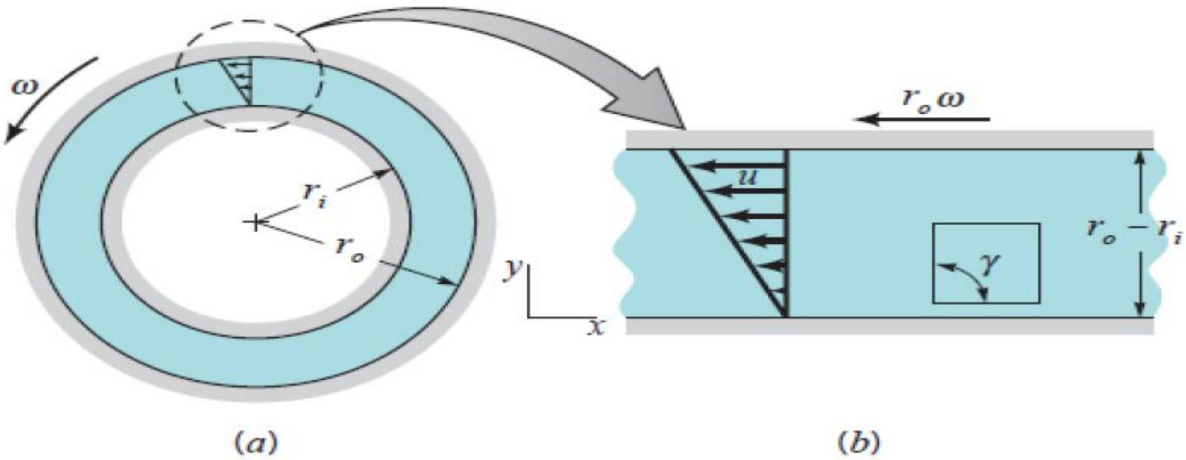

Fig. 1

3. A two-dimensional flow field for a nonviscous, incompressible fluid is described by the velocity components $u = U_0 + 2y$ $v = 0$ where U_0 is a constant. If the pressure at the origin (Fig. 2) is p_0, determine an expression for the pressure at (a) point A, and (b) point B. Explain clearly how you obtained your answer. Assume that the units are consistent and body forces may be neglected.

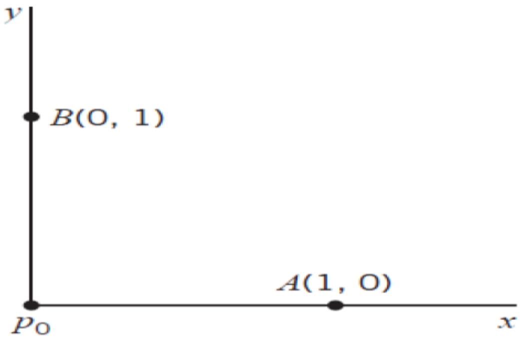

Fig. 2

1.2. Flow acceleration and substantial derivative

1. A two-dimensional velocity field is given by V=(x^2 - y^2 + x) **i** – (2xy + y) **j** in arbitrary units. At (*x*, *y*) = (1, 2), compute (*a*) the accelerations *ax* and *ay*, (*b*) the velocity component in the direction θ = 40°, (*c*) the direction of maximum velocity, and (*d*) the direction of maximum acceleration.

2. Consider a sphere of radius R immersed in a uniform stream U0, as shown in Fig.3. The fluid velocity along streamline AB is given by V = u**i** = U0 (1+R^3/x^3)**i** Find (`a) the
position of maximum fluid acceleration along AB and (b) the time required for a fluid particle to travel from A to B.

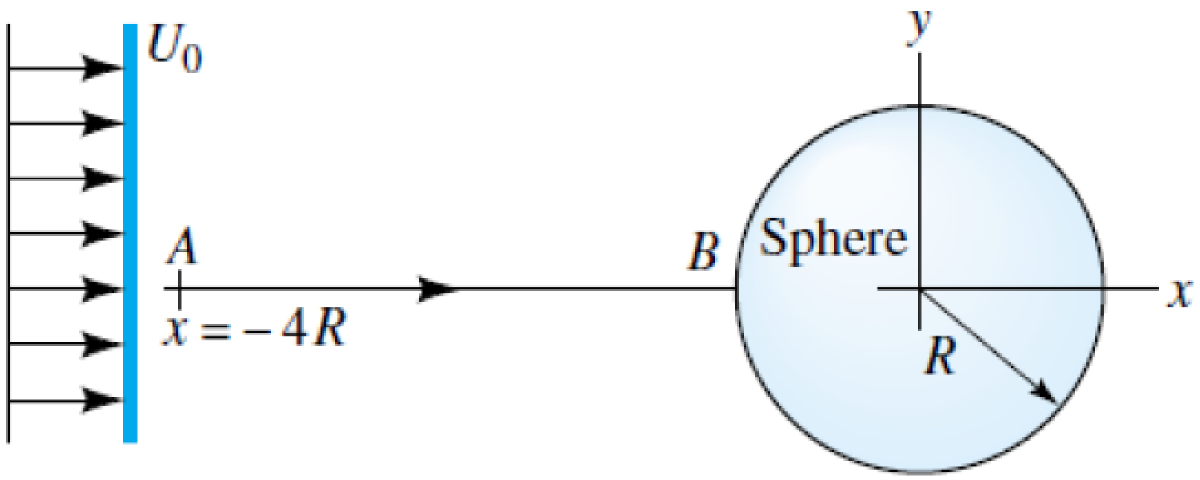

Fig. 3

3. When a valve is opened, fluid flows in the expansion duct of Fig. 4 according to the approximation
V = **i***U* (1-x/(2L)) tanh (Ut/L)
Find (a) the fluid acceleration at (x, t) = (L, L/U) and (b) the time for which the fluid acceleration at x = L is zero. Why does the fluid acceleration become negative after condition (b)?

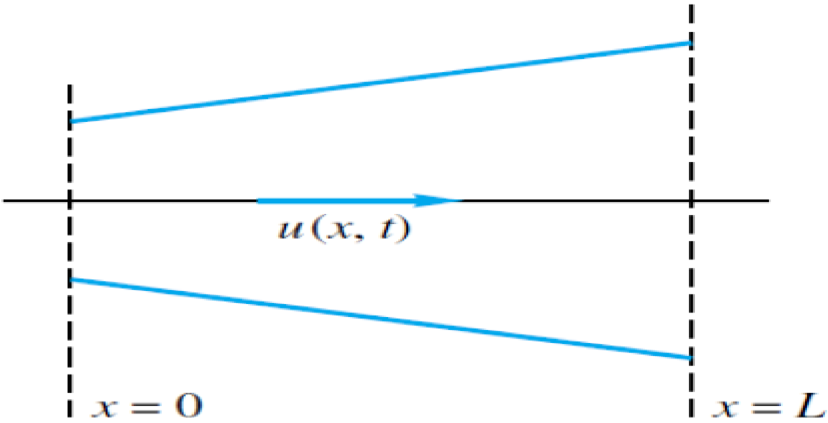

Fig. 4

4.2. Continuity

1. A piston compresses gas in a cylinder by moving at constant speed ***Y***, as in Fig. 5. Let the gas density and length at $t = 0$ be ρ_0 and L_0, respectively. Let the gas velocity vary linearly from $u = V$ at the piston face to $u = 0$ at $x = L$. If the gas density varies only with time, find an expression for $\rho(t)$.

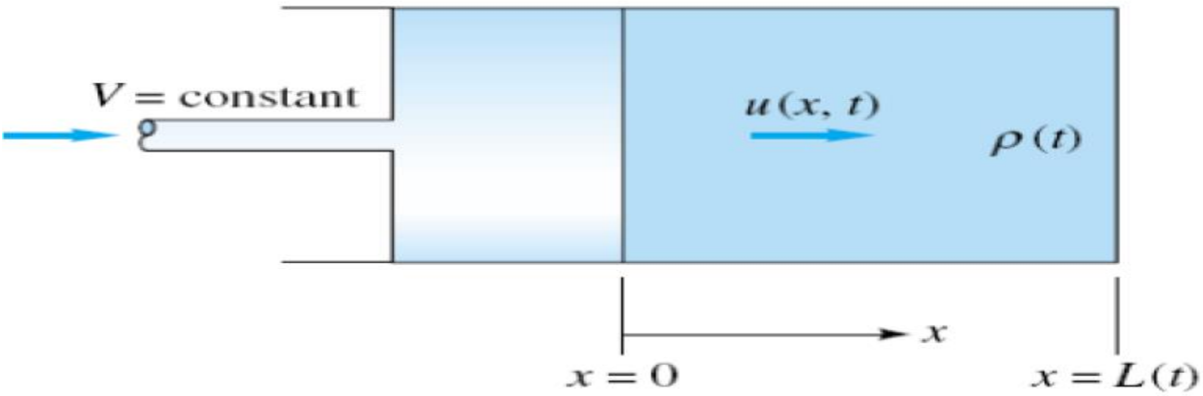

Fig. 5

2. A tank volume V contains gas at conditions (ρ_0, p_0, T_0). At time $t = 0$ it is punctured by a small hole of area A. According to the theory, the mass flow out of such a hole is approximately proportional to A and to the tank pressure. If the tank temperature is assumed constant and the gas is ideal, find an expression for the variation of density within the tank. 3. A velocity field is given by V = ($3y^2$- $3x^2$)**i** + Cxy**j** + 0**k**. Determine the value of the constant C if the flow is to be (a) incompressible and (b) irrotational.

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