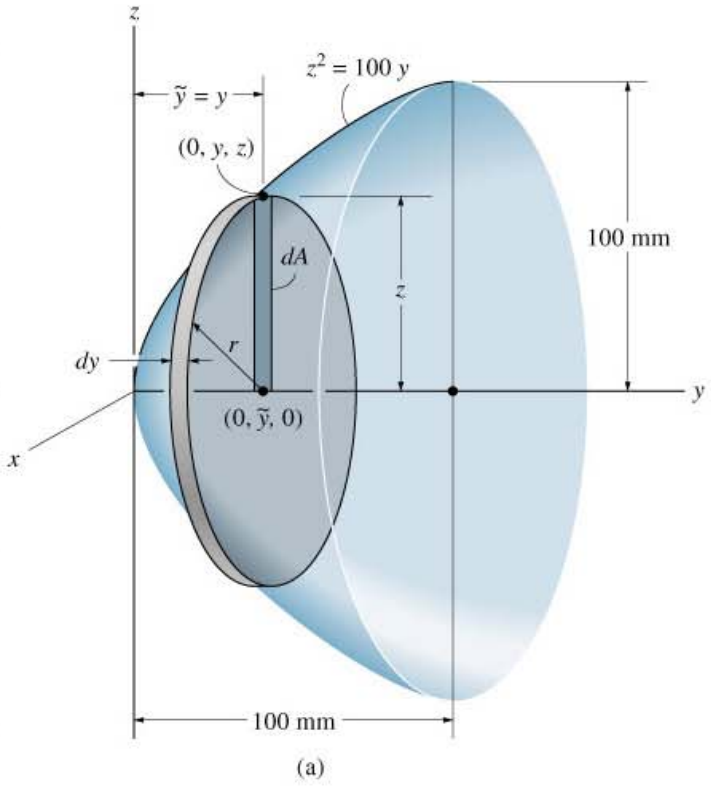
۱۷,۲,۲۴ بسمه تعالی



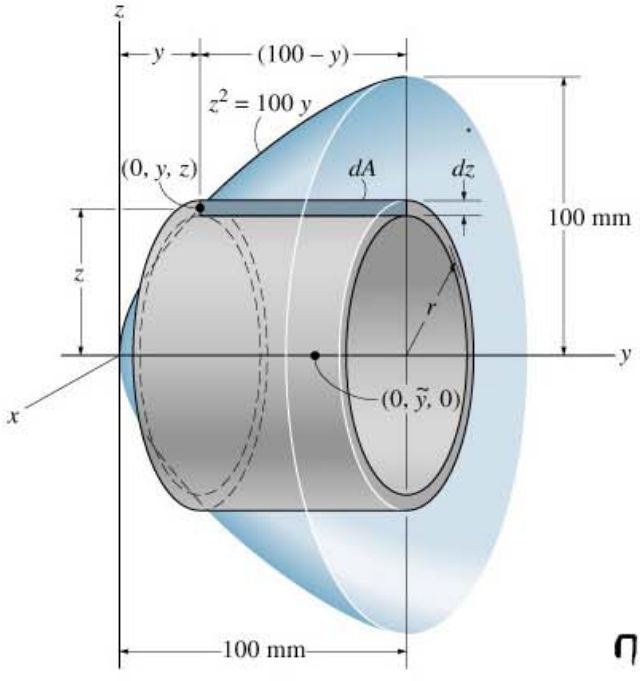
(a)

$$\bar{y} = \frac{\int \tilde{y}\, dV}{\int dV}$$

$$dV = \pi z^2\, dy \qquad \tilde{y} = y$$

$$\bar{y} = \frac{\int y(\pi z^2)\, dy}{\int \pi z^2 dy} = \frac{100\pi \int_0^{100} y^2 dy}{100\pi \int_0^{100} y\, dy}$$

$$\frac{\frac{y^3}{3}\Big|_0^{100}}{\frac{y^2}{2}\Big|_0^{100}} = \frac{\frac{100^3}{3}}{\frac{100^2}{2}} = \frac{200}{3} = 66.67 \text{ mm}$$



(b)

$$dV = 2\pi r\, dA = 2\pi z(100 - y)\, dz$$

$$\tilde{y} = y + \frac{100 - y}{2} = \frac{y + 100}{2}$$

$$\bar{y} = \frac{\int \frac{y+100}{2} \times 2\pi z(100 - y)\, dz}{\int 2\pi z(100 - y)\, dz}$$

$$\pi \int \left(\frac{z^2}{100} + 100\right) z \left(100 - \frac{z^2}{100}\right) dz =$$

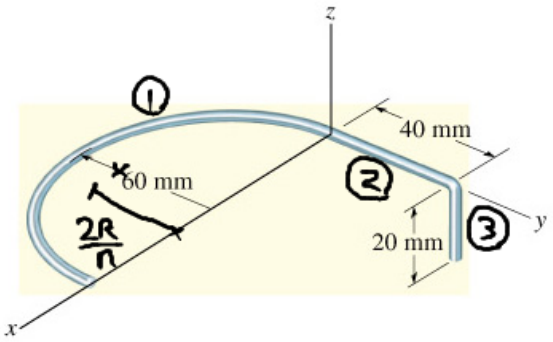
$$\pi \int_0^{100} z\left(100^2 - \frac{z^4}{100^2}\right) dz = \pi\left(\frac{100^2}{2} z^2 - \frac{z^6}{6 \times 100^2}\right)\Big|_0^{100}$$

$$\int \tilde{y}\, dV = \pi\left(\frac{100^4}{2} - \frac{100^4}{6}\right) = \frac{\pi \times 10^8}{3} \text{ mm}^4$$

$$\int dV = 2\pi \int_0^{100} z\left(100 - \frac{z^2}{100}\right) dz = 2\pi\left(\frac{100}{2} z^2 - \frac{z^4}{4 \times 100}\right)\Big|_0^{100} = 2\pi \times 100^3$$

$$\bar{y} = \frac{\frac{\pi \times 100^4}{3}}{\frac{2\pi \times 100^3}{4}} = \frac{4 \times 100}{3 \times 2} = \frac{200}{3} = 66.67 \text{ mm}$$

مرکز هندسی یا مرکز ثقل اجسام مرکب




(a)

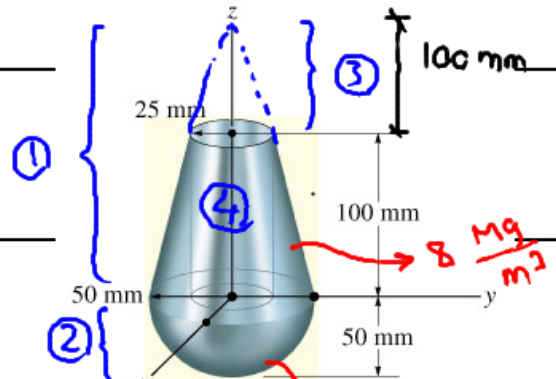
$$\bar{x} = \frac{\sum \tilde{x}_i L_i}{\sum L_i}$$ → $dA$, $dV$, $dm, dW$

| شئ | $L_i$ | $\tilde{x}$ | $\tilde{y}$ | $\tilde{z}$ | $\tilde{x}_i L_i$ | $\tilde{y}_i L_i$ | $\tilde{z}_i L_i$ |
|---|---|---|---|---|---|---|---|
| 1 | $60\pi$ | 60 | $-\frac{2\times 60}{\pi}$ | 0 | $3600\pi$ | $-7200$ | 0 |
| 2 | 40 | 0 | 20 | 0 | 0 | 800 | 0 |
| 3 | 20 | 0 | 40 | $-10$ | 0 | 800 | $-200$ |
| | $60+60\pi$ | | | | $3600\pi$ | $-5600$ | $-200$ |

$$\bar{x} = \frac{3600\pi}{60+60\pi} = 45.5 \text{ mm}$$

$$\bar{y} = \frac{-5600}{60+60\pi} = -22.5 \text{ mm}$$

$$\bar{z} = \frac{-200}{60+60\pi} = -.805 \text{ mm}$$




$$\bar{z} = \frac{\sum \tilde{z}_i V_i}{\sum V_i} \qquad \bar{z} = \frac{\sum \tilde{z}_i m_i}{\sum m_i}$$

| شئ | $m_i$ $V_i \times \rho_i$ | $\tilde{z}_i$ | $m_i$ $\tilde{z}_i V_i$ |
|---|---|---|---|
| | $1.008 \times \frac{1}{3}\pi \times 50^2 \times 200$ | $\frac{200}{4}$ | $.008 \times 26179939$ |

$\rho = 4 \frac{Mg}{m^3}$ (a)

$2 \cdot 0^{-4}x \frac{2}{3} \pi \times 50^3 \quad -\frac{3}{8} \times 50 \cdot 0^{-4}y - 4908739$

$3 \cdot 0^{-8}x - \frac{1}{3} \times \pi \times 25^2 \times 100 \quad 100 + \frac{100}{4} \cdot 0^{-8}x - 8181231$

$4 \cdot 0^{-8}x - \pi \times 25^2 \times 100 \quad 50 \cdot 0^{-8}x - 9817477$

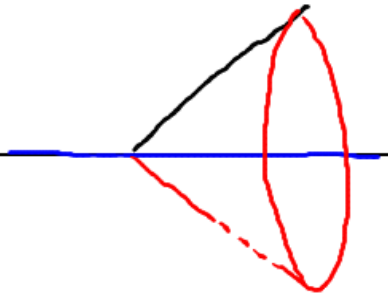
$523599 \qquad 3272492$

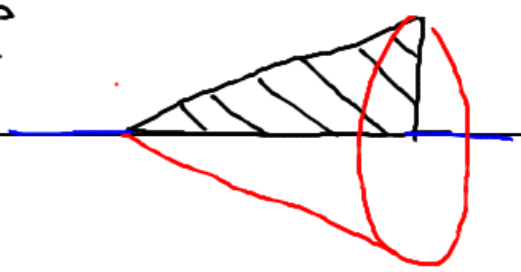
$\bar{z} = 6.25$ mm مرکز جرم $\qquad \bar{z} = 14.6$ mm مرکز جرم

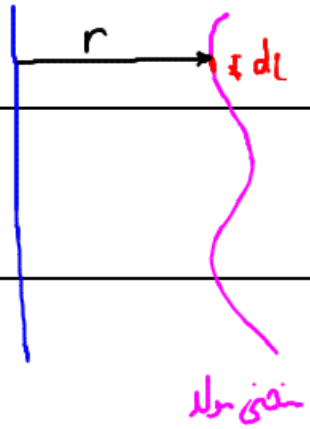
## قضیه پاپوس و گلدینوس :

محاسبه سطح جسم دورانی

حجم








محاسبه سطح جسم دورانی :

$$A = \int 2\pi r \, dL = 2\pi \int r \, dL = 2\pi \bar{r} L$$

$\theta \bar{r} L$

$\bar{r}$ ← مرکز منحنی مولد

$L$ ← طول منحنی مولد

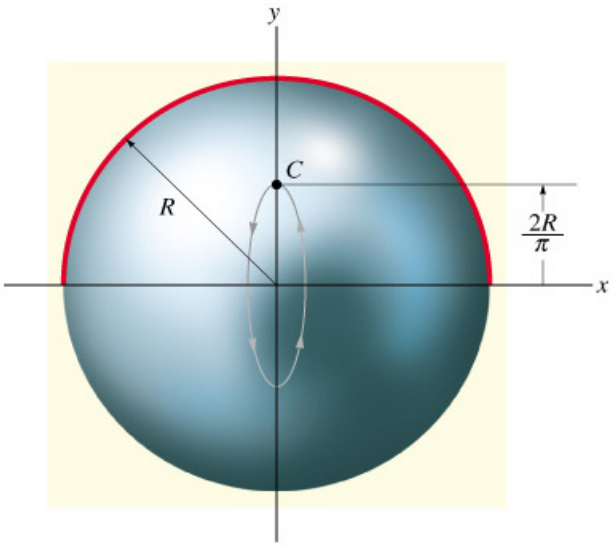
محاسبه حجم دورانی


$$V = \int 2\pi r \, dA = 2\pi \bar{r} A$$

$\theta \bar{r} A$

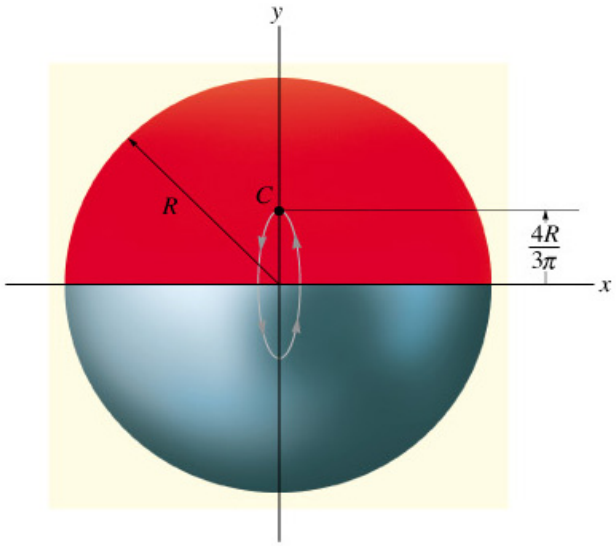
مسافتی است که مرکز سطح مولد در دوران طی کرده است

سطح مولد



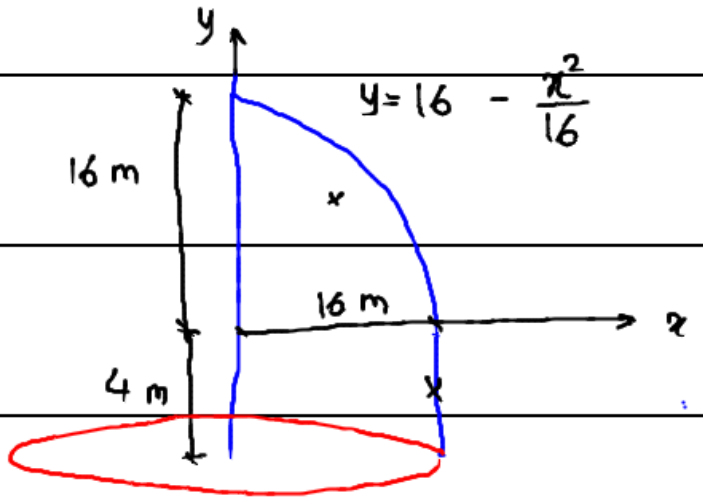

(a)

$$A = 2\pi \times \frac{2R}{\pi} \times \pi R = 4\pi R^2$$




(b)

$$V = 2\pi \times \frac{4R}{3\pi} \times \frac{\pi R^2}{2} = \frac{4\pi R^3}{3}$$




A =