

# Internal and string stability analyses of longitudinal platoon of vehicles with communication delay and actuator lag under constant spacing policy

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## Abstract

This paper studies the longitudinal control of a group of vehicles following a lead vehicle. A neighbor based upper level controller is proposed by considering communication delay and actuator lag. Constant spacing policy is used between successive vehicles. Two different approaches based on Lyapunov-Razumikhin and Lyapuniv-Krasovskii theorems are presented to stability analysis of closed loop dynamic. By simulation studies, it will be shown that the second approach is less conservatism than the first one. We consider the bidirectional leader following (BDLF) topology for inter-vehicle communication. Based on this structure, some sufficient conditions assuring string stability of platoon is derived. At the end of paper, four different scenarios are presented to study the robustness of algorithm against communication delay, actuator lag, disturbance, heterogeny and communication losses.

**Keywords:** *Platoon of vehicles; Communication delay; String stability; Internal stability; Actuator lag.*

## 1. Introduction

In recent decades, traffic congestion as an important social, economical and environmental problem, has received much attention [1-3]. An intelligent transportation system (ITS) is a possible solution to reduce the impact of traffic flow, improve safety and increase fuel efficiency [4,5]. The idea of ITS is to increase the traffic capacity by controlling vehicles to form platoons in which vehicles are maintained in small distance from each other.

In recent decades, the control problem of autonomous vehicles platoon has received much attention [6-8]. The early studies on platooning control have been carried out in 1990s [9]. The control structure of a vehicle has a hierarchical architecture. 1) the upper level controller and 2) the lower level controller. The upper level control calculates the desired acceleration of vehicle. The lower level one determines the desired inputs for throttle and brake actuators required to track the desired acceleration by utilizing the vehicle dynamic models, engine structure and nonlinear control techniques [10,11,12]. The vehicle and its internal

mechanisms play the role of node dynamics for upper level controller. The upper level control of vehicles platoon has been investigated in previous studies extensively [8,13-15].

In addition to internal stability, string stability in vehicle platooning should be considered. The string stability of a platoon refers to a property that spacing errors not to amplify as they propagate through the vehicles string [16]. Two prevalent spacing policies of controlling vehicles in a platoon are 1) constant spacing policy [17] and 2) constant time headway (CTH) spacing policy [18]. In CTH spacing policy, the desired inter vehicle distances are functions of velocity but in the other policy, they are constant values. There is a very important relation between string stability and spacing policies. For example, in the bidirectional communication structure, the string stability cannot be guaranteed by constant spacing policy.

There are several possible communication structures between vehicles in a platoon. If vehicles utilize only onboard sensors such as radar, the only variables available for the upper level controller are relative position and velocity [13,19] and the only possible communication structures are bidirectional

and predecessor following. On the other hand, if there exist wireless networks in platoon, each vehicle can communicate with other vehicles especially with the lead vehicle [20,21]. Numerous communication topologies are presented in [21].

To date, a great deal of research work has been done in vehicle platooning control. In general, there are two main approaches in stability analysis and control design of vehicular platoon. Analyses in 1) frequency domain [10,13,20,25] and 2) state space [26-31]. In [10], by considering lag of actuator, a decentralized adaptive cruise control (ACC) is designed to provide string stability and results are verified by experimental studies. The cluster treatment of characteristic roots (CTCR) is used in [13] to reveal the stabilizing parametric regions to render internal and string stability. A double integrator longitudinal dynamic for a homogenous platoon of vehicle under disturbance is proposed in [20]. In considered vehicular network, each agent only communicates with the lead and predecessor vehicles. Despite of disturbance and network communication delay, the proposed centralized controller certifies string stability [20]. A robust control is derived to stabilize vehicular network against data loss and transmission delay in a heterogeneous platoon [26]. A robust adaptive sliding mode control is proposed in [27] by adding online estimation of drag coefficients, vehicle mass and rolling resistance. A wireless based communication network is considered in [28] and by using Lyapunov-Krasovskii theorem some conditions assuring internal stability against multiple delays are derived.

Time delay is a prevalent phenomenon in multi-agent systems [32-35]. Time delay resulting from communication links have been also attracted much attention to vehicular platoon [10,20,25,28,29,36]. Time delay is considered homogenous in some studies [10,20] while it is assumed heterogenous in some others [28,29,30]. In [29] Lyapunov-Razumikhin formula is exploited to perform stability analysis by considering upper bounds for time varying communication delays. In [36] effect of information delay on string and internal stability for different frameworks is studied. It is shown that the ranges of controller parameters are smaller than those without considering the information delay.

The design of coupling control laws guaranteeing that each agent in a team converges to a consistent view of their information states is a known problem as consensus. In some studies consensus is solved under communication time delay either homogenous or heterogeneous [28,32,33,35]. To the best of our knowledge, there are very few results on consensus of second order multi agent systems under

both communication delays and lag of actuator. The objective of this paper is longitudinal control design of homogenous vehicular platoon by considering it as second order consensus problem with communication delay and actuator lag. The main contributions of this paper are as follows:

1. Solving platooning as a second order consensus problem by involving both communication time delays and actuator lag in internal and string stability.

2. Stability analysis of the proposed consensus problem by Lyapunov-Razumikhin approach.

3. The first approach renders very small bounds of communication delays guaranteeing internal stability. To obviate this defect, in the continuance, a new approach based on Lyapunov-Krasovskii functional and linear matrix inequality (LMI) techniques is presented. Simulation results indicate that the second approach is less conservatism than the first one.

In both methods, the constant spacing policy is considered between successive vehicles and a second order model is considered to describe the longitudinal motion of platoon.

The rest of this paper is organized as follows: in section 2, a brief introduction on graph theory and some mathematical preliminaries are presented. In section 3, a second order model describing the longitudinal dynamic of platoon is presented and a neighbor based coupling control law is defined. In continuance, two important theorems are presented to analyze the stability of closed loop dynamic. Section 4 presents some analyses of string stability. Some simulation studies are available in section 5 to verify the effectiveness of proposed approaches. Finally, this paper is concluded in section 6.

## 2. Graph theory and mathematical preliminaries

If each vehicle in the platoon network is regarded as a node, then their communication topology is described by a graph (basic concepts of graph theory can be found in [38,39]). Let  $G = (V, \mathcal{E}, \mathbf{W})$  is a weighted directed graph (digraph) of order  $N$  with  $V = \{1, 2, \dots, N\}$  representing node set.  $\mathcal{E} \subseteq V \times V$  is the set of edges and  $\mathbf{W}$  is the adjacency matrix of graph  $G$  with nonnegative elements. Each edge denoted as  $(i, j)$  means that node  $j$  can access information of node  $i$ , but not necessarily vice versa. Accordingly, node  $i$  is a neighbor of node  $j$ .  $N_i = \{j \in V : (j, i) \in \mathcal{E}, j \neq i\}$  is the neighbor set of node  $i$ . A graph is undirected if  $(i, j) \in \mathcal{E}$  implies

that  $(j, i) \in \mathcal{E}$ . A directed path is a sequence of connected edges in a graph. If there exists a path from node  $i$  to node  $j$ , node  $j$  is reachable from node  $i$ . An undirected graph  $G$  is connected if there exists a path between any two distinct nodes. Node  $i$  is globally reachable, if it is reachable from every other node in  $G$ . A digraph  $G$  is strongly connected if any two distinct nodes are reachable from each other.

The topology of a graph can be represented by two matrices: 1) the weighted adjacency matrix  $\mathbf{W} = [w_{ij}] \in \mathbb{R}^{N \times N}$  with  $w_{ij} > 0$  if  $(j, i) \in \mathcal{E}$  and  $w_{ij} = 0$ , otherwise and 2) the Laplacian matrix  $\mathbf{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$  with  $l_{ii} = \sum_{j=1, j \neq i}^N a_{ij}$  and  $l_{ij} = -w_{ij}, i \neq j$ .

For a multi-agent system, an agent is called a leader if it has no neighbor. An agent is called a follower if it has at least one neighbor. To study the leader-follower problem, another graph  $\bar{G}$  is considered associated with the system of  $N$  agent and one leader numbered by 0. Similarly, a diagonal matrix  $\mathbf{Z} \in \mathbb{R}^{N \times N}$  is defined as a leader adjacency matrix associate with  $\bar{G}$  with diagonal elements  $z_i = a_{i0}$ . If agent 0 is neighbor of agent  $i$ ,  $a_{i0} > 0$  and  $a_{i0} = 0$  otherwise. We say node 0 is globally reachable in  $\bar{G}$  if there is at least one path from every node  $i \in V$  to it. For  $\bar{G}$  we define an important matrix  $\mathbf{H} = \mathbf{L} + \mathbf{Z}$  which will be used in stability analysis in the next section.

**Lemma 1.** [40]: Matrix  $\mathbf{H}$  is positive definite if and only if node 0 is globally reachable in  $\bar{G}$ .

**Lemma 2.** [41]: For any symmetric matrix  $\bar{\mathbf{M}} = \begin{pmatrix} \bar{\mathbf{M}}_{11} & \bar{\mathbf{M}}_{12} \\ \bar{\mathbf{M}}_{12}^T & \bar{\mathbf{M}}_{22} \end{pmatrix}$ ,  $\bar{\mathbf{M}} \succ 0$  if and only if  $\bar{\mathbf{M}}_{11} \succ 0$  and  $\bar{\mathbf{M}}_{22} - \bar{\mathbf{M}}_{12}^T \bar{\mathbf{M}}_{11}^{-1} \bar{\mathbf{M}}_{12} \succ 0$ .

**Lemma 3.** [42,43]: For any vectors  $\mathbf{a}, \mathbf{b}$  and any positive definite matrix  $\boldsymbol{\zeta}$ , the inequality  $2\mathbf{a}^T \mathbf{b} \leq \mathbf{a}^T \boldsymbol{\zeta} \mathbf{a} + \mathbf{b}^T \boldsymbol{\zeta}^{-1} \mathbf{b}$  holds.

**Theorem 1 (Lyapunov-Razumikhin)** [24]: consider the following time delay system:

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}_t) \\ \mathbf{x}(\theta) = \boldsymbol{\varphi}(\theta), \theta \in [-\beta, 0] \end{cases} \quad (1)$$

where  $\mathbf{x}_t(\theta) = \mathbf{x}(t + \theta), \forall \theta \in [-\beta, 0]$  and  $\mathbf{f}(\mathbf{0}) = \mathbf{0}$ . Let  $\eta_1, \eta_2$  and  $\eta_3$  be continues, non-decreasing and nonnegative functions with  $\forall s > 0: \eta_1(s) > 0, \eta_2(s) > 0, \eta_3(s) > 0$  and  $\eta_1(0) = 0, \eta_2(0) = 0$ . Suppose that function  $\mathbf{f}: C([-r, 0], \mathbb{R}^n) \rightarrow \mathbb{R}^n$  maps the bounded sets of  $C([-r, 0], \mathbb{R}^n)$  into bounded sets of  $\mathbb{R}^n$ . If there is a continues function  $V(\mathbf{x}, t)$  such that

$$\eta_1(\|\mathbf{x}\|) \leq V(\mathbf{x}, t) \leq \eta_2(\|\mathbf{x}\|) \quad t \in \mathbb{R}, \mathbf{x} \in \mathbb{R}^n \quad (2a)$$

and there exists a continues non-decreasing function  $\eta(s)$  with  $\forall s > 0: \eta(s) > 0$  such that

$$\dot{V}(\mathbf{x}, t) \leq -\eta_3(\|\mathbf{x}\|) \quad (2b)$$

when

$V(\mathbf{x}(t + \theta), t + \theta) < \eta(V(\mathbf{x}, t)), \theta \in [-\beta, 0]$ , then the origin  $\mathbf{x} = \mathbf{0}$  is uniformly asymptotically stable.  $V(\mathbf{x}, t)$  is called Lyapunov-Razumikhin function if it satisfies conditions (2a) and (2b).

### 3. 3. Problem formulation

The consensus problem of a homogenous platoon of  $N+1$  vehicles which consists of a leader (numbered by 0) and  $N$  followers is investigated as shown in Fig.1.

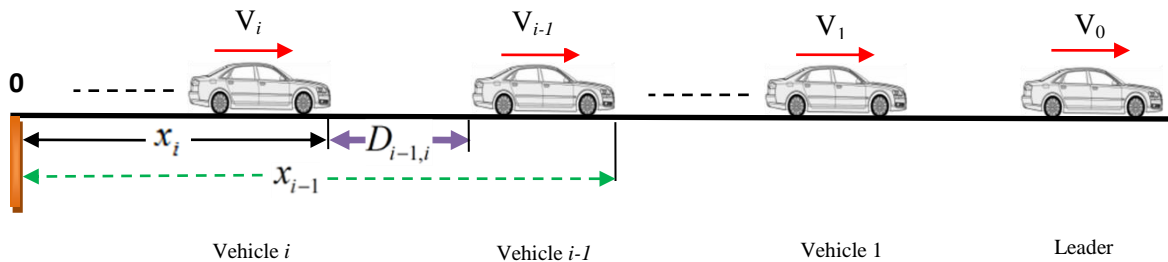


Fig1.: Longitudinal platoon of vehicles

The objective is keeping a desired constant inter vehicle distance between each two consecutive vehicles. A second order model is used to describe the longitudinal motion of vehicles:

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = \frac{1}{M} u_i \end{cases} \quad (3)$$

where  $x_i$  and  $v_i$  denote the position and velocity of  $i$ th vehicle in platoon,  $M$  is the mass of each vehicle and  $u_i$  is the control input. Because of communication delay, each vehicle cannot access the information from others immediately. So, the neighbor based coupling control law is expressed in the following form:

$$u_i(t) = k \sum_{j=1}^{N_i} w_{ij} [x_j(t-\tau) - x_i(t-\tau) - d_{ji}] + k z_i [x_0(t-\tau) - x_i(t-\tau) - d_{0i}] + D(v_0 - v_i) \quad (4)$$

where  $x_0$  and  $v_0$  are position and velocity of lead vehicle,  $\tau$  is the communication delay,  $k$  and  $D$  are control gains and  $N_i$  is the number of neighbors of  $i$ th vehicle. In order to achieve the constant spacing policy, it is necessary to add an offset term to control law. We consider that all vehicles follow the leader with a constant distance between consecutive vehicles.  $d_{ij}$  is the offset of vehicle  $i$  with vehicle  $j$

and  $d_{0i}$  is defined as  $d_{0i} = \sum_{j=1}^i (D_{j-1,j} + L_{j-1})$

where  $D_{i,i-1}$  is desired space between vehicles  $i$  and  $i-1$  and  $L_i$  is the length of vehicle  $i$ . It is assumed that leader velocity is partially constant. For vehicle  $i$  desired trajectory is defined as

$$x_i^d = x_0 - \sum_{j=1}^i (D_{j-1,j} + L_{j-1}) \quad (5)$$

By defining the tracking error as

$$e_i = x_i - x_i^d \Rightarrow \dot{e}_i = \dot{x}_i - \dot{x}_i^d \Rightarrow \ddot{e}_i = \ddot{x}_i \quad (6)$$

The control law (4) can be rewritten as

$$u_i(t) = k \sum_{j=1}^{N_i} w_{ij} [e_j(t-\tau) - e_i(t-\tau)] - k z_i e_i(t-\tau) - D \dot{e}_i(t) \quad (7)$$

Lag is an inevitable inherent property in many mechanical actuators such as vehicle engine [10]. Due to lag of engine, controller commands will not be available immediately. In the other words, the engine torque could not achieve the desired value instantly when the control signal is sent by the upper level controller. It means that the term of the control law  $u_i(t)$  is replaced by  $u_i(t - \Delta)$  where  $\Delta$  is the lag of

engine. With some practical experiments, the amount of  $\Delta$  is measured for a kind of vehicle in [10].

Considering lag of engine and replacing (7) in (3), we have

$$\ddot{e}_i = k \sum_{j=1}^{N_i} w_{ij} [e_j(t-\tau) - e_i(t-\tau)] - k z_i e_i(t-\tau) - D \dot{e}_i(t) \quad (8)$$

where

$$\tau_1 = \tau + \Delta, \tau_2 = \Delta, \bar{D} = \frac{1}{M} D, \bar{z}_i = \frac{1}{M} z_i, \bar{w}_{ij} = \frac{1}{M} w_{ij}$$

By defining the error vector  $\mathbf{e} = [e_1, \dots, e_N, e_1, \dots, e_N]^T$ , the closed loop dynamic of platoon can be represented as

$$\dot{\mathbf{e}} = \mathbf{A}\mathbf{e} + \mathbf{B}\mathbf{e}(t - \tau_1) + \mathbf{C}\mathbf{e}(t - \tau_2) \quad (9)$$

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{I}_N \\ \mathbf{0} & \mathbf{0} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ -k\mathbf{H}/M & \mathbf{0} \end{pmatrix}, \mathbf{C} = \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -D\mathbf{I}_N/M \end{pmatrix}$$

**Theorem 2.** If the following conditions hold, the linear time delay system (9) is globally asymptotically stable.

1- Lead vehicle is globally reachable in the platoon for all communication topologies.

2- There exists a symmetric positive definite matrix  $\bar{\mathbf{P}}$  satisfying the following Lyapunov equation  $\bar{\mathbf{P}}\mathbf{H} + \mathbf{H}^T\bar{\mathbf{P}} = \mathbf{I}_N$  (10)

$$3- k < \frac{2(D\bar{k}_2 - M)}{\bar{k}_2^2} \frac{\gamma}{\mu}, \text{ where}$$

$$\gamma = \min(\text{eig}(\bar{\mathbf{P}})), \mu = \max\{\text{eig}[\bar{\mathbf{P}}\mathbf{H}\mathbf{H}^T\bar{\mathbf{P}}]\}, \bar{k}_1 = \frac{D}{M}, \bar{k}_2 > \frac{M}{D} \quad (11)$$

**Proof:** According to Lyapunov theorem, there exists a positive definite matrix  $\bar{\mathbf{P}} \in \mathfrak{R}^{N \times N}$  satisfying (10) if  $\mathbf{H}$  is positive stable. According to lemma 1,  $\mathbf{H}$  is positive definite if the lead vehicle is globally reachable in the platoon. By using Newton-Leibnitz formula, we can write

$$\begin{aligned} \mathbf{e}(t - \tau_1) &= \mathbf{e}(t) - \int_{-\tau_1}^0 \dot{\mathbf{e}}(t+s) ds = \mathbf{e}(t) - \mathbf{A} \int_{-\tau_1}^0 \mathbf{e}(t+s) ds - \mathbf{B} \int_{-\tau_1}^{-\tau_1} \mathbf{e}(t+s) ds - \mathbf{C} \int_{-\tau_1}^{-\tau_2} \mathbf{e}(t+s) ds \\ \mathbf{B}^2 &= \mathbf{B}\mathbf{C} = \mathbf{0} \Rightarrow \mathbf{B}\mathbf{e}(t - \tau_1) = \mathbf{B}\mathbf{e}(t) - \mathbf{B}\mathbf{A} \int_{-\tau_1}^0 \mathbf{e}(t+s) ds \\ \mathbf{e}(t - \tau_2) &= \mathbf{e}(t) - \int_{-\tau_2}^0 \dot{\mathbf{e}}(t+s) ds = \mathbf{e}(t) - \mathbf{A} \int_{-\tau_2}^0 \mathbf{e}(t+s) ds - \mathbf{B} \int_{-\tau_2}^{-\tau_2} \mathbf{e}(t+s) ds - \mathbf{C} \int_{-\tau_2}^{-\tau_2} \mathbf{e}(t+s) ds \\ \mathbf{C}\mathbf{A} &= \mathbf{0} \Rightarrow \mathbf{C}\mathbf{e}(t - \tau_2) = \mathbf{C}\mathbf{e}(t) - \mathbf{C}\mathbf{B} \int_{-\tau_2}^{-\tau_2} \mathbf{e}(t+s) ds - \mathbf{C}^2 \int_{-\tau_2}^{-\tau_2} \mathbf{e}(t+s) ds \end{aligned} \quad (12)$$

By using (12) and (13), we can write (9) in the form

$$\dot{\mathbf{e}} = \mathbf{E}\mathbf{e}(t) - \mathbf{B}\mathbf{A} \int_{-\tau_1}^0 \mathbf{e}(t+s) ds - \mathbf{C}\mathbf{B} \int_{-\tau_1}^{-\tau_2} \mathbf{e}(t+s) ds - \mathbf{C}^2 \int_{-\tau_2}^{-\tau_2} \mathbf{e}(t+s) ds \quad (14)$$

where  $\mathbf{E} = \mathbf{A} + \mathbf{B} + \mathbf{C}$ . Consider the following Lyapunov-Razumikhin function

$$V = \mathbf{e}^T \mathbf{P} \mathbf{e}, \mathbf{P} = \begin{pmatrix} \bar{k}_1 \bar{\mathbf{P}} & \bar{\mathbf{P}} \\ * & \bar{k}_2 \bar{\mathbf{P}} \end{pmatrix}, \quad \bar{k}_1 = \frac{D}{M}, \bar{k}_2 > \frac{M}{D} \quad (15)$$

Time derivative of  $V$  along (14) leads to where

$$\mathbf{Q} = -(\mathbf{E}^T \mathbf{P} + \mathbf{P} \mathbf{E}) = \begin{pmatrix} \frac{k}{M} \mathbf{I}_N & \frac{k \bar{k}_2}{M} \mathbf{H}^T \bar{\mathbf{P}} \\ * & 2(\frac{D}{M} \bar{k}_2 - 1) \bar{\mathbf{P}} \end{pmatrix} \quad (17)$$

According to lemma (2),  $\mathbf{Q} \succ \mathbf{0}$  if the following condition holds

$$2(\frac{D}{M} \bar{k}_2 - 1) \bar{\mathbf{P}} - \frac{k \bar{k}_2^2}{M} \bar{\mathbf{P}} \mathbf{H} \mathbf{H}^T \bar{\mathbf{P}} \succ \mathbf{0} \quad (18)$$

in other words

$$2(D \bar{k}_2 - M) \bar{\mathbf{P}} - k \bar{k}_2^2 \bar{\mathbf{P}} \mathbf{H} \mathbf{H}^T \bar{\mathbf{P}} \succ \mathbf{0} \Rightarrow 2(D \bar{k}_2 - M) \gamma > k \bar{k}_2^2 \mu \quad (19)$$

$$\Rightarrow k < \frac{2(D \bar{k}_2 - M) \gamma}{\bar{k}_2^2 \mu}$$

$$\gamma = \min(\text{eig}(\bar{\mathbf{P}})), \mu = \max\{\text{eig}[\bar{\mathbf{P}} \mathbf{H} \mathbf{H}^T \bar{\mathbf{P}}]\}$$

By using lemma (3) for three times with the following choices

$$\mathbf{a} = \mathbf{A}^T \mathbf{B}^T \mathbf{P} \mathbf{e}, \mathbf{B}^T \mathbf{C}^T \mathbf{P} \mathbf{e}, \mathbf{C}^T \mathbf{P} \mathbf{e}; \quad \mathbf{b} = \mathbf{e}(t+s); \quad \boldsymbol{\varsigma} = \mathbf{P}^{-1} \quad (20)$$

from (16) we can write

$$\dot{V} < -\mathbf{e}^T \mathbf{Q} \mathbf{e} + \tau_1 \mathbf{e}^T \mathbf{P} \mathbf{B}^T \mathbf{A} \mathbf{P}^{-1} \mathbf{A}^T \mathbf{B}^T \mathbf{P} \mathbf{e} + \int_{-\tau_1}^0 \mathbf{e}^T(t+s) \mathbf{P} \mathbf{e}(t+s) ds + \tau_1 \mathbf{e}^T \mathbf{P} \mathbf{C} \mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T \mathbf{C}^T \mathbf{P} \mathbf{e} + \int_{-\tau_2}^{-\tau_1} \mathbf{e}^T(t+s) \mathbf{P} \mathbf{e}(t+s) ds + \tau_2 \mathbf{e}^T \mathbf{P} \mathbf{C}^2 \mathbf{P}^{-1} (\mathbf{C}^2)^T \mathbf{P} \mathbf{e} + \int_{-2\tau_2}^{-\tau_2} \mathbf{e}^T(t+s) \mathbf{P} \mathbf{e}(t+s) ds$$

Take  $\eta(s) = qs$  for some constant  $q > 1$ . In the case of  $V(\mathbf{x}(t+s)) < qV(\mathbf{x}(t)) - \max(2\tau_2, \tau_1 + \tau_2) \leq s \leq 0$  we have:

By assuming known value for  $\tau_2$ , if  $\tau_1$  satisfies the following inequality

$$\tau_1 < \frac{\lambda_{\min}}{\|\mathbf{P} \mathbf{B} \mathbf{A} \mathbf{P}^{-1} \mathbf{A}^T \mathbf{B}^T \mathbf{P}\| + \|\mathbf{P} \mathbf{C} \mathbf{B} \mathbf{P}^{-1} \mathbf{B}^T \mathbf{C}^T \mathbf{P}\| + 2q \|\mathbf{P}\|}, \quad \lambda_{\min} = \min(\text{eig}(-\mathbf{Q} + \tau_2 (\mathbf{P} \mathbf{C}^2 \mathbf{P}^{-1} (\mathbf{C}^2)^T \mathbf{P} + q \mathbf{P}))) \quad (21)$$

$$\text{then } \dot{V} < -\bar{\eta} \mathbf{e}^T \mathbf{e}, \quad \bar{\eta} > 0.$$

**Remark 1.** As it will be shown in simulation study, this approach is very conservatism presenting small bounds for  $\tau_1$  and  $\tau_2$ . In continuance a new approach which is less conservatism based on Lyapunov-Krasovskii functional is presented.

**Theorem 2.** The linear time delay system (9) is globally asymptotically stable if there exist symmetric matrices

$$\mathbf{P}, \mathbf{Q}_i, \mathbf{S}_i, \quad i = 1, 2, \quad \mathbf{X} = \begin{pmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} \\ * & \mathbf{X}_{22} \end{pmatrix} \succ \mathbf{0}, \mathbf{Y} = \begin{pmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ * & \mathbf{Y}_{22} \end{pmatrix} \succ \mathbf{0}, \mathbf{X}_{ij},$$

$\mathbf{Y}_{ij} \in \mathbb{R}^{N \times N}$  and arbitrary matrices  $\mathbf{M}_1, \mathbf{M}_2, \mathbf{N}_1, \mathbf{N}_2$  such that the following expressions satisfy:

$$\mathbf{P} \succ \mathbf{0}, \quad \mathbf{Q}_i \succ \mathbf{0}, \quad \mathbf{S}_i \succ \mathbf{0}, \quad i = 1, 2 \quad (22)$$

$$\boldsymbol{\Psi} = \begin{pmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} & \mathbf{N}_1 \\ * & \mathbf{X}_{13} & \mathbf{N}_2 \\ * & * & \mathbf{S}_1 \end{pmatrix} \succ \mathbf{0}, \boldsymbol{\xi} = \begin{pmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{M}_1 \\ * & \mathbf{Y}_{13} & \mathbf{M}_2 \\ * & * & \mathbf{S}_2 \end{pmatrix} \succ \mathbf{0}, \boldsymbol{\Phi} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} & \boldsymbol{\Phi}_{13} \\ * & \boldsymbol{\Phi}_{22} & \boldsymbol{\Phi}_{23} \\ * & * & \boldsymbol{\Phi}_{33} \end{pmatrix} \prec \mathbf{0}$$

3)

where

$$\boldsymbol{\Phi}_{11} = \mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} + \sum_{i=1}^2 \mathbf{Q}_i + \sum_{i=1}^2 \tau_i \mathbf{A}^T \mathbf{S}_i \mathbf{A} + \sum_{i=1}^2 (\mathbf{N}_i + \mathbf{M}_i) + \tau_1 \mathbf{X}_{11} + \tau_2 \mathbf{Y}_{11}$$

$$\boldsymbol{\Phi}_{12} = \mathbf{P} \mathbf{B} + \sum_{i=1}^2 \tau_i \mathbf{A}^T \mathbf{S}_i \mathbf{B} + \mathbf{N}_2^T - \mathbf{N}_1 + \tau_1 \mathbf{X}_{12}$$

$$\boldsymbol{\Phi}_{13} = \mathbf{P} \mathbf{C} + \sum_{i=1}^2 \tau_i \mathbf{A}^T \mathbf{S}_i \mathbf{C} + \mathbf{M}_2^T - \mathbf{M}_1 + \tau_2 \mathbf{Y}_{12}, \quad \boldsymbol{\Phi}_{22} = -\mathbf{Q}_1 + \sum_{i=1}^2 \tau_i \mathbf{B}^T \mathbf{S}_i \mathbf{B} - \mathbf{N}_2^T - \mathbf{N}_2 + \tau_1 \mathbf{X}_{22}$$

$$\boldsymbol{\Phi}_{23} = \sum_{i=1}^2 \tau_i \mathbf{B}^T \mathbf{S}_i \mathbf{C}, \quad \boldsymbol{\Phi}_{33} = -\mathbf{Q}_2 + \sum_{i=1}^2 \tau_i \mathbf{C}^T \mathbf{S}_i \mathbf{C} - \mathbf{M}_2^T - \mathbf{M}_2 + \tau_2 \mathbf{Y}_{22}$$

**Proof.** Consider the following Lyapunov-Krasovskii functional

$$V = \mathbf{e}^T \mathbf{P} \mathbf{e} + \sum_{i=1}^2 \int_{t-\tau_i}^t \mathbf{e}^T \mathbf{Q}_i \mathbf{e} ds + \sum_{i=1}^2 \int_{-\tau_i}^0 \int_{t+\theta}^t \dot{\mathbf{e}}^T \mathbf{S}_i \dot{\mathbf{e}} ds d\theta \quad (24)$$

Time derivative of (24) along (9) is

$$\dot{V} = [\mathbf{A} \mathbf{e} + \mathbf{B} \mathbf{e}_1 + \mathbf{C} \mathbf{e}_2]^T \mathbf{P} \mathbf{e} + \mathbf{e}^T \mathbf{P} [\mathbf{A} \mathbf{e} + \mathbf{B} \mathbf{e}_1 + \mathbf{C} \mathbf{e}_2] + \mathbf{e}^T \mathbf{Q}_1 \mathbf{e} - \mathbf{e}_1^T \mathbf{Q}_1 \mathbf{e}_1 + \mathbf{e}^T \mathbf{Q}_2 \mathbf{e} - \mathbf{e}_2^T \mathbf{Q}_2 \mathbf{e}_2 + \tau_1 [\mathbf{A} \mathbf{e} + \mathbf{B} \mathbf{e}_1 + \mathbf{C} \mathbf{e}_2]^T \mathbf{S}_1 [\mathbf{A} \mathbf{e} + \mathbf{B} \mathbf{e}_1 + \mathbf{C} \mathbf{e}_2] + \tau_1 [\mathbf{A} \mathbf{e} + \mathbf{B} \mathbf{e}_1 + \mathbf{C} \mathbf{e}_2]^T \mathbf{S}_2 [\mathbf{A} \mathbf{e} + \mathbf{B} \mathbf{e}_1 + \mathbf{C} \mathbf{e}_2] - \int_{-\tau_1}^0 \dot{\mathbf{e}}^T(t+\theta) \mathbf{S}_1 \dot{\mathbf{e}}(t+\theta) d\theta - \int_{-\tau_2}^0 \dot{\mathbf{e}}^T(t+\theta) \mathbf{S}_2 \dot{\mathbf{e}}(t+\theta) d\theta$$

where  $\mathbf{e}_1 = \mathbf{e}(t - \tau_1)$ ,  $\mathbf{e}_2 = \mathbf{e}(t - \tau_2)$ . By adding the following obvious terms to (25)

$$2[\mathbf{e}^T \mathbf{N}_1 + \mathbf{e}_1^T \mathbf{N}_2][\mathbf{e} - \mathbf{e}_1 - \int_{-\tau_1}^0 \dot{\mathbf{e}}(t+\theta) d\theta] = 0, \quad 2[\mathbf{e}^T \mathbf{M}_1 + \mathbf{e}_2^T \mathbf{M}_2][\mathbf{e} - \mathbf{e}_2 - \int_{-\tau_2}^0 \dot{\mathbf{e}}(t+\theta) d\theta] = 0$$

$$\tau_1 \delta_1^T \mathbf{X} \delta_1 - \int_{-\tau_1}^0 \delta_1^T \mathbf{X} \delta_1 d\theta = 0, \quad \tau_2 \delta_2^T \mathbf{X} \delta_2 - \int_{-\tau_2}^0 \delta_2^T \mathbf{X} \delta_2 d\theta = 0, \quad \delta_1 = [\mathbf{e}^T, \mathbf{e}_1^T]^T, \quad \delta_2 = [\mathbf{e}^T, \mathbf{e}_2^T]^T$$

and some simplifications, (25) can be expressed as

$$\dot{V} = \boldsymbol{\delta}^T \boldsymbol{\Phi} \boldsymbol{\delta} - \int_{-\tau_1}^0 \boldsymbol{\delta}_1^T \boldsymbol{\Psi} \boldsymbol{\delta}_1 d\theta - \int_{-\tau_2}^0 \boldsymbol{\delta}_2^T \boldsymbol{\xi} \boldsymbol{\delta}_2 d\theta, \quad \boldsymbol{\delta} = [\mathbf{e}^T, \mathbf{e}_1^T, \mathbf{e}_2^T]^T, \quad \boldsymbol{\delta}_3 = [\mathbf{e}^T, \mathbf{e}_1^T, \dot{\mathbf{e}}^T]^T, \quad \boldsymbol{\delta}_4 = [\mathbf{e}^T, \mathbf{e}_2^T, \dot{\mathbf{e}}^T]^T$$

If conditions (22) and (23) hold,  $\dot{V} < 0$ . So, the closed loop system (9) is globally asymptotically stable.

#### 4. String stability

Communication structure: the lead vehicle is equipped to onboard integration sensor and a GPS receiver allows it to measure own absolute position and velocity. Each follower is equipped to radar sensors enable it to measure relative position with respect to predecessor and subsequent vehicles and a wireless to communicate with the lead vehicle. Fig. 2 shows the proposed topology structure.



Fig2. Each Bi-directional leader follower topology

So, the closed loop dynamic of  $i$ th vehicle is in the following form

$$\ddot{e}_i = \bar{k}_{i,j-1}[e_{i-1}(t-\tau_1) - e_i(t-\tau_1)] + \bar{k}_{i,j+1}[e_{i+1}(t-\tau_1) - e_i(t-\tau_1)] - \bar{k}_i e_i(t-\tau_1) - \bar{D}\dot{e}_i(t-\tau_2) \quad (27)$$

where

$$\bar{k}_{i,j-1} = \frac{k}{M} w_{i,j-1}, \bar{k}_{i,j+1} = \frac{k}{M} w_{i,j+1}, \bar{k}_i = \frac{k}{M} z_i, \bar{D} = \frac{D}{M}$$

Taking the Laplace transformation of both sides of (27), leads to

$$E_i = G_{i-1}E_{i-1} + G_{i+1}E_{i+1}$$

$$G_{i-1} = \frac{\bar{k}_{i,j-1}e^{-\tau_1 s}}{s^2 + s\bar{D}e^{-\tau_2 s} + (\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i)e^{-\tau_1 s}}, G_{i+1} = \frac{\bar{k}_{i,j+1}e^{-\tau_1 s}}{s^2 + s\bar{D}e^{-\tau_2 s} + (\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i)e^{-\tau_1 s}} \quad (28)$$

where  $E_i(s)$  is the Laplace transform of  $e_i(t)$ . Applying algebraic manipulations, we have

$$\frac{E_i}{E_{i-1}} = \frac{G_{i-1}}{1 - G_{i+1} \frac{E_{i+1}}{E_i}} \quad (29)$$

The key idea in string stability of a platoon is to avoid that spacing errors are amplified upstream the platoon. A platoon is string stable if any disturbance on the lead vehicle motion is attenuated along the rest of string.

**Theorem 4.** Consider the longitudinal motion of a homogenous platoon of vehicles with bidirectional leader following communication topology. If the following conditions hold, the platoon is string stable under time delay and actuator lag.

$$(k_{i,j-1} + k_{i,j+1} + k_i)(\tau_1 + \tau_2 + \sqrt{(\tau_1 + \tau_2)^2 + \frac{2M}{k_{i,j-1} + k_{i,j+1} + k_i}}) \leq D \leq \frac{M}{2\tau_2}$$

$$k_i \geq |k_{i,j+1} - k_{i,j-1}|, \quad i = 1, \dots, N, \quad k_{N,N+1} = 0, \quad k_{1,0} = 0. \quad (30)$$

where

$$k_{i,i-1} = M\bar{k}_{i,i-1}, \quad k_{i,i+1} = M\bar{k}_{i,i+1}, \quad k_i = M\bar{k}_i$$

**Proof.** It is inferred from (29) that if the

$$\text{conditions } |G_{i-1}|, |G_{i+1}| < 0.5 \quad \text{and} \quad \left| \frac{E_N}{E_{N-1}} \right| \leq 1$$

$$\left| \frac{E_i}{E_{i-1}} \right| \leq 1 \quad \text{then} \quad |G_{i-1}| = \sqrt{\frac{p_{i-1}}{q_{i-1}}} \leq \frac{1}{2} \Rightarrow q_{i-1} - 4p_{i-1} \geq 0. \quad \text{Consider}$$

$$q_{i-1} = \omega^4 + \omega^2 \bar{D}^2 + (\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i)^2 - 2\omega^2 \bar{D} \sin \tau_2 \omega - 2(\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i) \omega^2 \cos \tau_1 \omega + 2(\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i) \omega \bar{D} \sin(\tau_1 + \tau_2) \omega$$

$$p_{i-1} = \bar{k}_{i,j-1}$$

$$(31)$$

By some calculations,  $q_{i-1} - 4p_{i-1} \geq 0$  is simplified to

$$(\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i)^2 - 4(\bar{k}_{i,j-1})^2 + 2(\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i) \omega \bar{D} \sin(\tau_1 + \tau_2) \omega - 2(\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i) \omega^2 \cos \tau_1 \omega - 2\omega^3 \bar{D} \sin \tau_2 \omega + \omega^4 + \omega^2 \bar{D}^2 \geq 0 \quad (32)$$

According to

$$\forall \bar{\alpha} \geq 0: \sin \bar{\alpha} \leq \bar{\alpha} \rightarrow -\sin \bar{\alpha} \geq -\bar{\alpha}, \cos \bar{\alpha} \leq 1 \rightarrow -\cos \bar{\alpha} \geq -1 \quad (33)$$

If the following conditions hold, (34) is valid.

$$1 - 2\bar{D}\tau_2 \geq 0, \quad \bar{D}^2 - 2(\bar{k}_{i,j-1} + \bar{k}_{i,j+1} + \bar{k}_i)(1 + \bar{D}(\tau_1 + \tau_2)) \geq 0, \quad k_i \geq k_{i,j-1} - k_{i,j+1} \quad (35)$$

Performing the same analysis for  $|G_{i+1}| = \sqrt{\frac{p_{i+1}}{q_{i+1}}} \leq \frac{1}{2}$ , redounds to

The closed loop dynamic of the  $N$ th vehicle is

$$\ddot{e}_N = \bar{k}_{N,N-1}[e_{N-1}(t-\tau_1) - e_N(t-\tau_1)] - \bar{k}_N e_N(t-\tau_1) - \bar{D}\dot{e}_N(t-\tau_2)$$

$$\left| \frac{E_N}{E_{N-1}} \right| \leq 1$$

Condition holds if

$$1 - 2\bar{D}\tau_2 \geq 0, \quad \bar{D}^2 - 2(\bar{k}_{N,N-1} + \bar{k}_N)(1 + \bar{D}(\tau_1 + \tau_2)) \geq 0 \quad (38)$$

By doing elementary calculations, we can show the constraints (35), (36) and (38) are same as following constraints.

5.

$$(k_{i,i-1} + k_{i,i+1} + k_i)(\tau_1 + \tau_2 + \sqrt{(\tau_1 + \tau_2)^2 + \frac{2M}{k_{i,i-1} + k_{i,i+1} + k_i}}) \leq D \leq \frac{M}{2\tau_2}$$

$$k_i \geq |k_{i,i+1} - k_{i,i-1}|, \quad i = 1, \dots, N, \quad k_{N,N+1} = 0, \quad k_{1,0} = 0.$$

### 5. Simulation study

A five vehicles platoon with bidirectional leader following topology (Fig. 2) is considered. The matrices  $\mathbf{W}$ ,  $\mathbf{Z}$  and  $\mathbf{H}$  for the proposed topology are as follows

$$\mathbf{W} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \mathbf{Z} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathbf{H} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

In all scenarios of this section, the velocity of lead vehicle is considered as Fig. 3.  $\delta_i = x_{i-1} - x_i - L_{i-1}$  is defined as spacing error in all simulations.

**Scenario 1.** In this scenario, platoon is considered homogenous. All parameters and controller gains are listed in Table 1.

For all two approaches,  $\tau_2$  is assumed known and  $\tau_1$  is calculated to keep  $\dot{V} < 0$ . By using data of Table 1, we obtain  $\gamma = 0.113$ ,  $\mu = 1.65$  and we also assume that  $q = 1.01$ . By solving LMIs (10), (22) and (23) the matrices  $\bar{\mathbf{P}}$  for the first approach and  $\mathbf{P}, \mathbf{Q}_1, \mathbf{Q}_2, \mathbf{S}_1, \mathbf{S}_2$  for the second approach are calculated as:

$$\bar{\mathbf{P}} = \begin{pmatrix} 0.3095 & 0.1190 & 0.0476 & 0.0238 \\ 0.1190 & 0.2381 & 0.0952 & 0.0476 \\ 0.0476 & 0.0952 & 0.2381 & 0.1190 \\ 0.0238 & 0.0476 & 0.1190 & 0.3095 \end{pmatrix}$$

$$\mathbf{P} = \begin{pmatrix} 0.1376 & -0.0009 & 0.0001 & -0.0002 & -0.0050 & -0.0014 & -0.0005 & -0.0001 \\ -0.0009 & 0.1387 & -0.0012 & 0.0001 & -0.0014 & -0.0041 & -0.0010 & -0.0005 \\ 0.0001 & -0.0012 & 0.1387 & -0.0009 & -0.0005 & -0.0010 & -0.0041 & -0.0014 \\ -0.0002 & 0.0001 & -0.0009 & 0.1376 & -0.0001 & -0.0005 & -0.0014 & -0.0050 \\ -0.0050 & -0.0014 & -0.0005 & -0.0001 & 0.0541 & 0.0097 & -0.0002 & -0.0006 \\ -0.0014 & -0.0041 & -0.0010 & -0.0005 & 0.0097 & 0.0442 & 0.0092 & -0.0002 \\ -0.0005 & -0.0010 & -0.0041 & -0.0014 & -0.0002 & 0.0092 & 0.0442 & 0.0097 \\ -0.0001 & -0.0005 & -0.0014 & -0.0050 & -0.0006 & -0.0002 & 0.0097 & 0.0541 \end{pmatrix}$$

$$\mathbf{Q}_1 = \begin{pmatrix} 0.1116 & -0.0015 & 0.0022 & -0.0003 & -0.0087 & -0.0000 & -0.0000 & -0.0000 \\ -0.0015 & 0.1154 & -0.0040 & 0.0022 & -0.0000 & -0.0087 & -0.0001 & -0.0000 \\ 0.0022 & -0.0040 & 0.1154 & -0.0015 & -0.0000 & -0.0001 & -0.0087 & -0.0000 \\ -0.0003 & 0.0022 & -0.0015 & 0.1116 & -0.0000 & -0.0000 & -0.0000 & -0.0087 \\ -0.0087 & -0.0001 & -0.0000 & -0.0000 & 0.0988 & -0.0015 & -0.0009 & -0.0002 \\ -0.0000 & -0.0087 & -0.0001 & -0.0000 & -0.0015 & 0.0993 & -0.0007 & -0.0009 \\ -0.0000 & -0.0001 & -0.0087 & -0.0000 & -0.0009 & -0.0007 & 0.0993 & -0.0015 \\ -0.0000 & -0.0000 & -0.0000 & -0.0087 & -0.0002 & -0.0009 & -0.0015 & 0.0988 \end{pmatrix}$$

$$\mathbf{Q}_2 = \begin{pmatrix} 0.0343 & 0.0000 & -0.0000 & 0.0000 & -0.0021 & -0.0008 & -0.0003 & -0.0001 \\ 0.0000 & 0.0343 & 0.0000 & -0.0000 & -0.0008 & -0.0017 & -0.0006 & -0.0003 \\ -0.0000 & 0.0000 & 0.0343 & 0.0000 & -0.0003 & -0.0006 & -0.0017 & -0.0008 \\ 0.0000 & -0.0000 & 0.0000 & 0.0343 & -0.0001 & -0.0003 & -0.0008 & -0.0021 \\ -0.0021 & -0.0008 & -0.0003 & -0.0001 & 0.1026 & 0.0371 & 0.0108 & 0.0011 \\ -0.0008 & -0.0017 & -0.0006 & -0.0003 & 0.0371 & 0.0763 & 0.0274 & 0.0108 \\ -0.0003 & -0.0006 & -0.0017 & -0.0008 & 0.0108 & 0.0274 & 0.0763 & 0.0371 \\ -0.0001 & -0.0003 & -0.0008 & -0.0021 & 0.0011 & 0.0108 & 0.0371 & 0.1026 \end{pmatrix}$$

$$\mathbf{S}_1 = \begin{pmatrix} 0.1603 & -0.0029 & -0.0004 & -0.0005 & -0.0053 & -0.0013 & -0.0003 & 0.0003 \\ -0.0029 & 0.1628 & -0.0030 & -0.0004 & -0.0013 & -0.0043 & -0.0008 & -0.0003 \\ -0.0004 & -0.0030 & 0.1628 & -0.0029 & -0.0003 & -0.0008 & -0.0043 & -0.0013 \\ -0.0005 & -0.0004 & -0.0029 & 0.1603 & 0.0003 & -0.0003 & -0.0013 & -0.0053 \\ -0.0053 & -0.0013 & -0.0003 & 0.0003 & 0.0159 & 0.0069 & 0.0008 & -0.0005 \\ -0.0013 & -0.0043 & -0.0008 & -0.0003 & 0.0069 & 0.0097 & 0.0057 & 0.0008 \\ -0.0003 & -0.0008 & -0.0043 & -0.0013 & 0.0008 & 0.0057 & 0.0097 & 0.0069 \\ 0.0003 & -0.0003 & -0.0013 & -0.0053 & -0.0005 & 0.0008 & 0.0069 & 0.0159 \end{pmatrix}$$

$$\mathbf{S}_2 = \begin{pmatrix} 0.1844 & -0.0001 & -0.0001 & -0.0000 & -0.0037 & 0.0006 & -0.0001 & 0.0001 \\ -0.0001 & 0.1844 & 0.0000 & -0.0001 & 0.0006 & -0.0044 & 0.0008 & -0.0001 \\ -0.0001 & 0.0000 & 0.1844 & -0.0001 & -0.0001 & 0.0008 & -0.0044 & 0.0006 \\ -0.0000 & -0.0001 & -0.0001 & 0.1844 & 0.0001 & -0.0001 & 0.0006 & -0.0037 \\ -0.0037 & 0.0006 & -0.0001 & 0.0001 & 0.0644 & 0.0197 & 0.0017 & -0.0015 \\ 0.0006 & -0.0044 & 0.0008 & -0.0001 & 0.0197 & 0.0464 & 0.0165 & 0.0017 \\ -0.0001 & 0.0008 & -0.0044 & 0.0006 & 0.0017 & 0.0165 & 0.0464 & 0.0197 \\ 0.0001 & -0.0001 & 0.0006 & -0.0037 & -0.0015 & 0.0017 & 0.0197 & 0.0644 \end{pmatrix}$$

Figs. 4 and 5 show the string stability of the platoon and velocity of vehicles, respectively. According to this Fig. 4, in the acceleration and deceleration motion periods of leader, the amplitude of error decreases upstream the platoon.

**Scenario 2.** In this scenario, a heterogeneous platoon is considered. Table 3 shows the parameters used in this scenario.

**Scenario 3.** Here, we analyze the string stability in presence of the sinusoidal disturbance  $d(t) = 0.5\sin(0.7t)$  acting on leader motion. All parameters are same as (Table 1). Figs. 8 and 9 show the string stability of the platoon and vehicles velocity in presence of disturbance signal. As Fig. 8 indicates, the platoon is string stable against disturbance signal.

**Scenario (4):** This scenario studies the effect of communication losses and switching topologies on the closed-loop stability. It is assumed that the communication topology varies between leader predecessor following (LPF), bidirectional (BD) and bidirectional leader following (BDLF), respectively. Figs. 10 and 11 show the performance of string stability and vehicles velocity in the platoon.

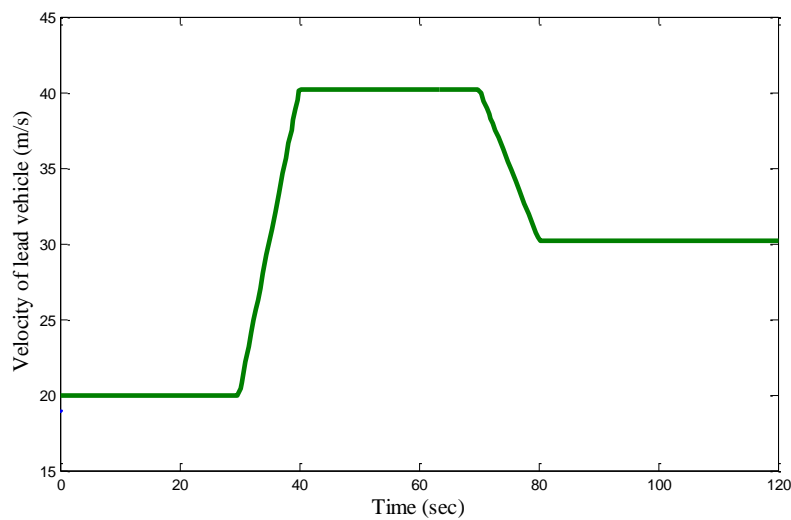


Fig3. velocity of lead vehicle

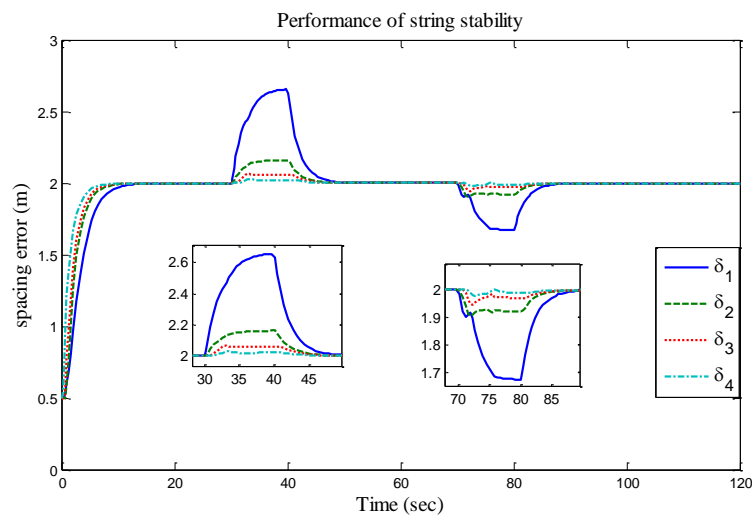


Fig4. Performance of string stability of the platoon in the first scenario

Table 1. Parameters of platoon and controller gains

Parameters	Description	Value
$M(Kg)$	Mass of vehicle	1600
$K$	Gain of controller	2100
$D$	Gain of controller	7200
$\tau_1(s)$	Time delay	$0.21 \sin(t) $
$\tau_2(s)$	Lag of motor	0.11
$L(m)$	Length of vehicle	4
$d_{i,i+1}(m)$	Desired distance between two consecutive vehicles	2



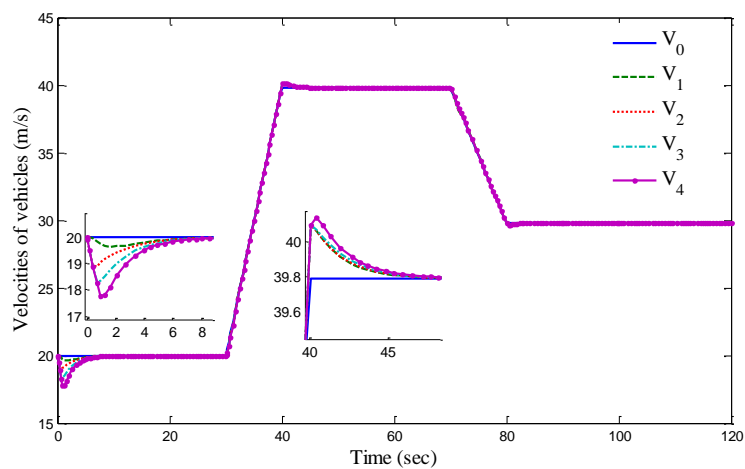


Fig. 5: Velocities of vehicles in the platoon

Table 3: Parameters of heterogeneous platoon in the second scenario

Vehicle parameters	M (Kg)	Lag of motor (s)	L (m)
1	1600	0.1	4
2	1800	0.12	3.6
3	1400	0.09	3.8
4	1500	0.11	4.2

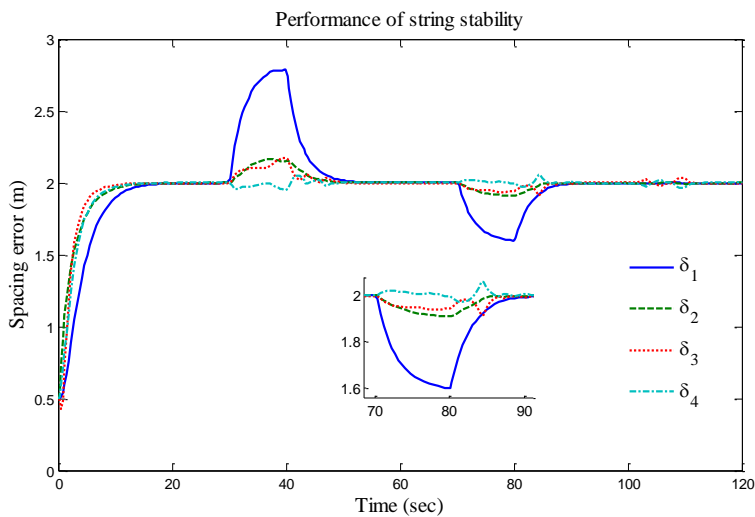


Fig. 6: Performance of string stability in second scenario.

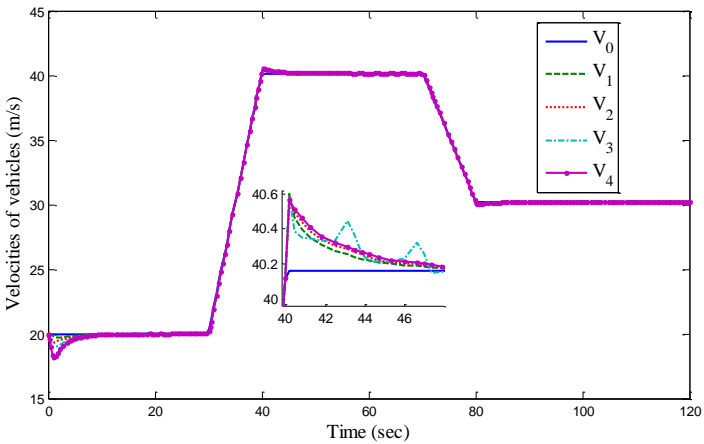


Fig. 7: Velocities of vehicles in platoon.

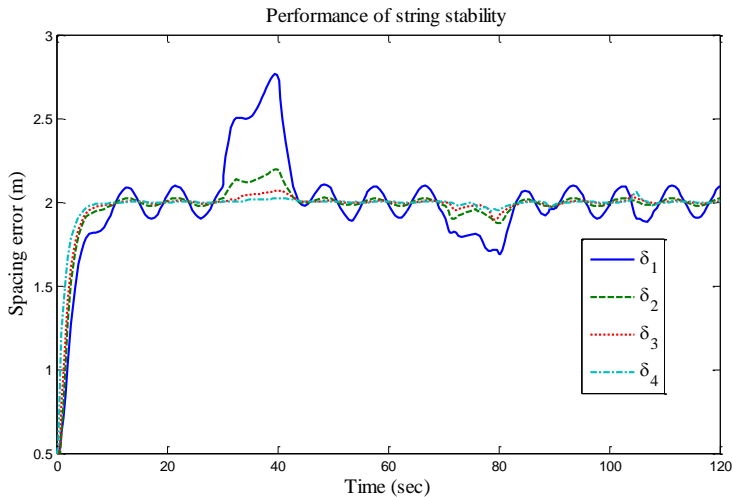


Fig. 8: String instability of platoon.

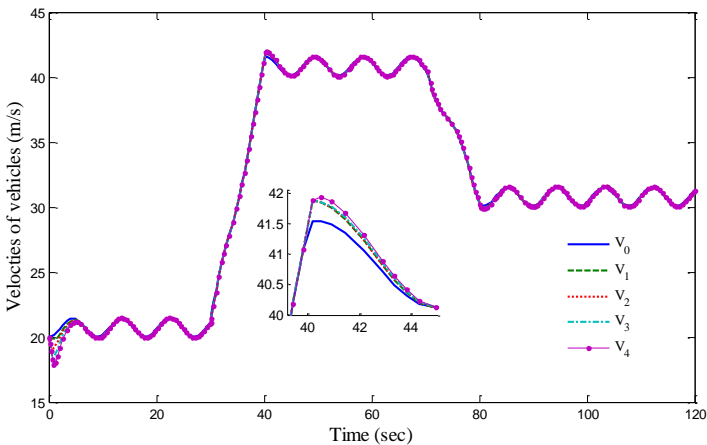


Fig. 9: Velocities of vehicles in 3th scenario

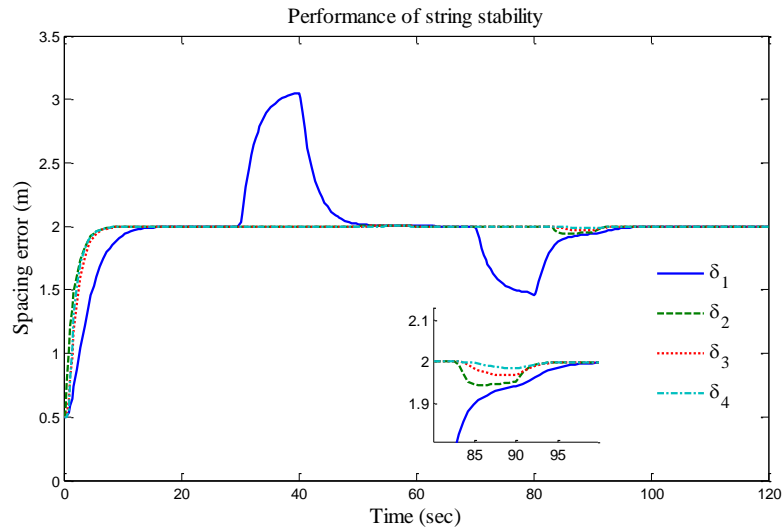


Fig. 10: Performance of string stability in scenario 4

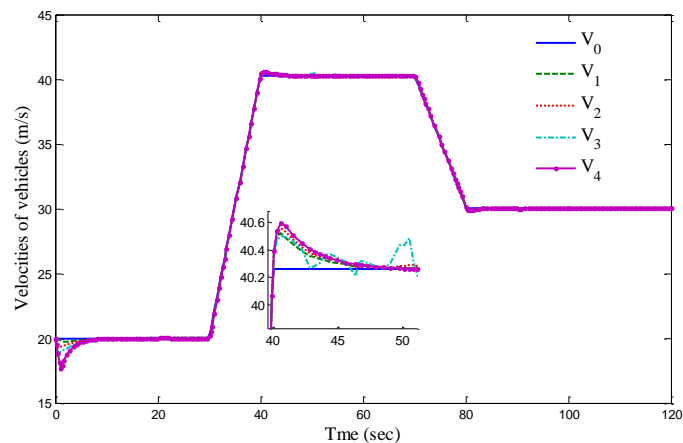


Fig. 11: Velocities of vehicles in scenario 4

## 6. Conclusion

This paper investigates the internal and string stability analyses of a platoon of homogenous vehicles with constant spacing policy between consecutive vehicles. Both communication delay and actuator lag are considered in the closed loop dynamic of vehicles network. Based on Lyapunov-Razumikhin and Lyapunov-Krasovskii theorems two approaches are presented to analyze the stability of closed loop system. By simulation studies it is shown that the second approach is less conservatism than the first one. Simulation studies with four different scenarios are presented to show the effectiveness of the proposed methods. The first scenario shows that despite of delays and lags, the homogenous vehicles platoon is internal and string stable. The second scenario investigates the heterogeneous platoon and it

shows a desired performance for internal and string stability. The third scenario illustrates the robustness of algorithm against disturbance acting the lead vehicle and finally, the fourth scenario studies the effect of switching topology and communication losses on internal and string stability. It is shown that the closed loop dynamic has suitable behavior in case of topology variances between LPF, BD and BDLF topologies.

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