



## LAYOUT OPTIMIZATION OF TRUSS STRUCTURES USING CENTER OF MASS OPTIMIZATION ALGORITHM

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### ABSTRACT

The main objective of this paper is to optimize the size and layout of planar truss structures simultaneously. To deal with this challenging type of truss optimization problem, the center of mass optimization (CMO) metaheuristic algorithm is utilized, and an extensive parametric study is conducted to find the best setting of internal parameters of the algorithm. The CMO metaheuristic is based on the physical concept of the center of mass in space. The effectiveness of the CMO metaheuristic is demonstrated through the presentation of three benchmark truss layout optimization problems. The numerical results indicate that the CMO is competitive with other metaheuristics and, in some cases, outperforms them.

**Keywords:** seismic performance level; steel moment resisting frame; neural network; feed-forward back-propagation.

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### 1. INTRODUCTION

One of the complex and challenging areas of structural optimization is optimum design involving sizing and layout variables. In many cases, traditional gradient-based techniques may not be sufficient to handle the additional complexity of these design optimization problems. The problem's complexity arises from considering variables of different natures simultaneously. It is necessary to optimize structural cross-sections and geometry simultaneously for layout optimization of structures with fixed topology. In these cases, a design space with large dimensions is encountered due to the large number of design variables consisting of cross-sectional areas and nodal coordinates. The selection of the cross-sections of the structural members from a discrete list of available sections leads to a discrete design space. In such discrete layout optimization problems, constraints on member

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tensile-compressive stresses and nodal displacements increase the possibility of being trapped in local optima [1-3]. Therefore, to solve truss layout optimization problems, it is necessary to use powerful optimization algorithms.

Over the past few decades, numerous metaheuristic algorithms have been proposed for structural optimization. These algorithms were inspired by different natural metaphors (such as evolution theory, biology, and physics) and have proved to be more effective and reliable than traditional gradient-based methods in solving complex and challenging optimization problems [4-8]. One of the physics-based metaheuristic algorithms is the Center of Mass Optimization (CMO) algorithm [9]. The principle behind the CMO is that mass must be balanced around its center of mass in space. The effectiveness of CMO in solving benchmark sizing optimization problems of truss structures and seismic performance-based design optimization of steel moment-resisting frames has been demonstrated in recent years compared to some metaheuristics [9-11]. Therefore, the present paper uses the CMO to address the layout optimization problem of trusses.

This paper presents three design examples of layout optimization for trusses with 15, 18, and 47 bars. For each design example, a sensitivity analysis is conducted to determine the optimal values of an internal parameter of the CMO. The numerical results reveal that the CMO is a competitive metaheuristic algorithm that, in some cases, even outperforms other techniques proposed in the literature.

## 2. LAYOUT OPTIMIZATION PROBLEM FORMULATION

The objective of layout optimization of trusses is usually to minimize their weight while taking into account certain design constraints. The design variables include the cross-sectional areas of the members and the coordinates of nodes in the structure. These variables are chosen from a set of discrete and continuous available values respectively. Therefore, the optimization problem can be formulated as follows

$$\text{Minimize: } w(X) = \sum_{i=1}^{ne} \rho_i A_i L_i \quad (1)$$

$$\text{Subject to: } \begin{cases} g_{s,i}(X) = \left( \frac{\sigma}{\sigma_{all}} \right)_i - 1 \leq 0, i = 1, 2, \dots, ne \\ g_{d,j}(X) = \left( \frac{d}{d_{all}} \right)_j - 1 \leq 0, j = 1, 2, \dots, nj \end{cases} \quad (2)$$

$$X = \{X_A \quad X_G\}^T \quad (3)$$

$$X_A = \{A_1 \quad A_2 \quad \dots \quad A_{ne}\}^T \in \Delta_A \quad (4)$$

$$X_G = \{G_1 \quad G_2 \quad \dots \quad G_{ng}\}^T \in \Delta_G \quad (5)$$

where  $w$  is the weight of the truss structure;  $X$  is the vector of design variables;  $\rho_i$ ,  $A_i$  and  $L_i$

are the weight density, cross-sectional area and the length of the  $i$ th member, respectively;  $g_s$ ,  $\sigma$  and  $\sigma_{all}$  are the stress constraint, stress and allowable stress of the  $i$ th member, respectively;  $g_d$ ,  $d$  and  $d_{all}$  are the displacement constraint, nodal displacement and allowable displacement of the  $j$ th node, respectively;  $ne$  and  $nj$  are the numbers of members and joints, respectively;  $X_A$  and  $X_G$  are the vectors of cross-sectional areas of members and joints coordinates related design variables, respectively;  $\Delta_A$  and  $\Delta_G$  are domains of cross-sectional areas and joints coordinates, respectively and  $ng$  is the number of joints coordinates.

In this paper, the constraints of the layout optimization problem are handled using the exterior penalty function method (EPFM) [12] in which the pseudo unconstrained objective function is expressed as follows

$$\Phi(X) = w(X) \left( 1 + r_p \sum_{k=1}^{nc} (\max\{0, g_k(X)\}) \right) \quad (6)$$

where  $\Phi$  is the pseudo unconstrained objective function;  $r_p$  is the penalty parameter and  $nc$  is the number of design constraints.

### 3. CENTER OF MASS OPTIMIZATION

CMO was proposed in [9] based on the concept of center of mass in physics. In the CMO algorithm, a population including  $np$  randomly selected particles ( $X_i$ ,  $i \in [1, np]$ ) is generated in design space. The mass of  $i$ th particle  $m_i$  is determined as follows

$$m_i = \frac{1}{f(X_i)} \quad (7)$$

Particles are sorted based on their mass values in ascending order and then they are equally divided into two groups G1 and G2. The first half of the particles are put in G1 and the others in G2. The particles in G1 are paired with their corresponding ones in G2. The position of the center of mass and the distance between  $j$ th ( $j=1, \dots, np/2$ ) pair of particles in iteration  $t$  are determined as follows

$$X_j^c(t) = \frac{m_j X_j(t) + m_{j+\frac{np}{2}} X_{j+\frac{np}{2}}(t)}{m_j + m_{j+\frac{np}{2}}} \quad (8)$$

$$d_j(t) = \left| X_j(t) - X_{j+\frac{np}{2}}(t) \right| \quad (9)$$

To switch between exploration and exploitation of the CMO algorithm, the following controlling parameter ( $CP$ ) is computed in which  $t_{max}$  is the maximum number of iterations and  $\alpha$  is a constant value.

$$CP(t) = \exp\left(-\frac{\alpha t}{t_{max}}\right) \quad (10)$$

The position of  $j$ th couple of particles is updated using the following equations

$$\text{if } d_j(t) > CP(t) \quad (11)$$

$$X_j(t+1) = X_j(t) - R_1 \left( X_j^C(t) - X_j(t) \right) + R_2 \left( X_b - X_j(t) \right) \quad (12)$$

$$X_{j+\frac{np}{2}}(t+1) = X_{j+\frac{np}{2}}(t) - R_3 \left( X_j^C(t) - X_{j+\frac{np}{2}}(t) \right) + R_4 \left( X_b - X_{j+\frac{np}{2}}(t) \right) \quad (13)$$

$$\text{if } d_j(t) \leq CP(t) \quad (14)$$

$$X_j(t+1) = X_j(t) + R_5 \left( X_j^C(t) - X_{j+\frac{np}{2}}(t) \right) \quad (15)$$

$$X_{j+\frac{np}{2}}(t+1) = X_{j+\frac{np}{2}}(t) + R_6 \left( X_j^C(t) - X_{j+\frac{np}{2}}(t) \right) \quad (16)$$

where  $R_1$  to  $R_6$  are vector of random numbers in  $[0,1]$ ; and  $X_b$  is the best solution found.

There is a mutation operator in CMO to decrease the probability of local optima entrapment. A mutation rate  $mr = 0.1$  is taken and in iteration  $t$  a number between 0 and 1 is randomly selected for each particle in group G1 ( $X_j, j=1, \dots, np/2$ ).

$$r_j(t) \in [0, 1] \quad (17)$$

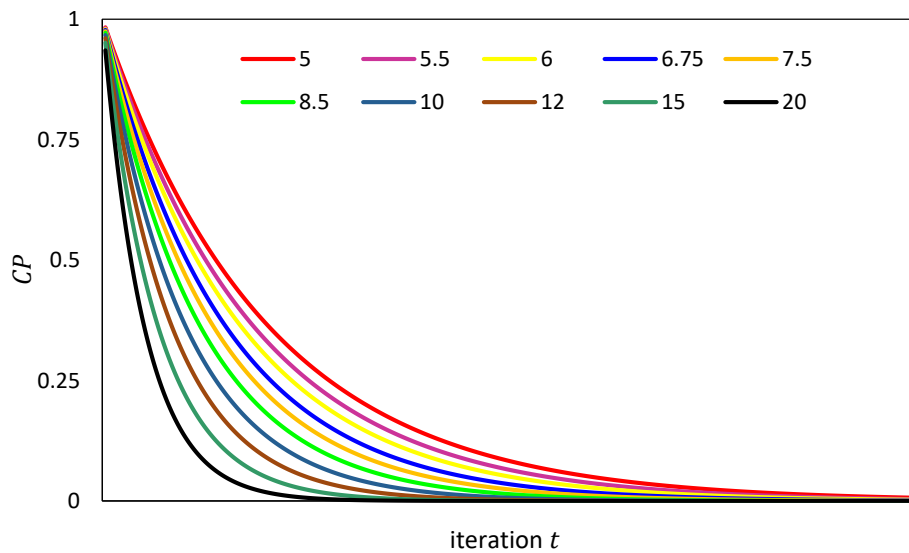
$$X_j(t) = \{x_{j1}(t) \quad x_{j2}(t) \quad \dots \quad x_{ji}(t) \quad \dots \quad x_{jm}(t)\}^T \quad (18)$$

For  $j$ th particle, if the selected random number is less than the mutation rate, one randomly selected component will be regenerated in the design space as follows

$$\text{if } r_j(t) \leq mr \rightarrow x_{ji}(t) = x_{ji}^L + \mu(t) \times (x_{ji}^U - x_{ji}^L) \quad (19)$$

where  $\mu$  is a random number in the interval  $[0, 1]$  in iteration  $t$ ; and  $x_{ij}^L$  and  $x_{ij}^U$  are lower and upper bounds of  $x_{ji}$  in design space.

The CMO metaheuristic consists of an internal parameter, namely  $\alpha$  which plays a crucial role in the convergence of the algorithm. Determining the optimal value of this parameter requires performing a sensitivity analysis. As, in the original CMO algorithm  $\alpha = 5.0$ , in this paper, different values of 5.0, 5.5, 6.0, 6.75, 7.5, 8.5, 10.0, 12.0, 15.0, and 20 are considered for  $\alpha$ . Fig. 1 depicts  $CP$  over  $t$  for the different values of  $\alpha$ .

Figure 1.  $CP - t$  graphs for different values of  $\alpha$ 

#### 4. NUMERICAL EXAMPLES

Three truss sizing-layout benchmark optimization problems including discrete sizing variables and continuous configuration variables are presented to investigate the performance of the CMO metaheuristic algorithm. The presented design examples in this paper are a planar 15-bar truss, a planar 18-bar truss, and a planar 47-bar truss.

##### 4.1 15-bar truss

The first design problem is a planar 15-bar truss shown in Fig. 2. A concentrated 10 kips load is applied to node 8 as shown in Fig. 2. The material density and the modulus of elasticity are  $0.1 \text{ lb/in}^3$  and  $10^4 \text{ ksi}$ , respectively.

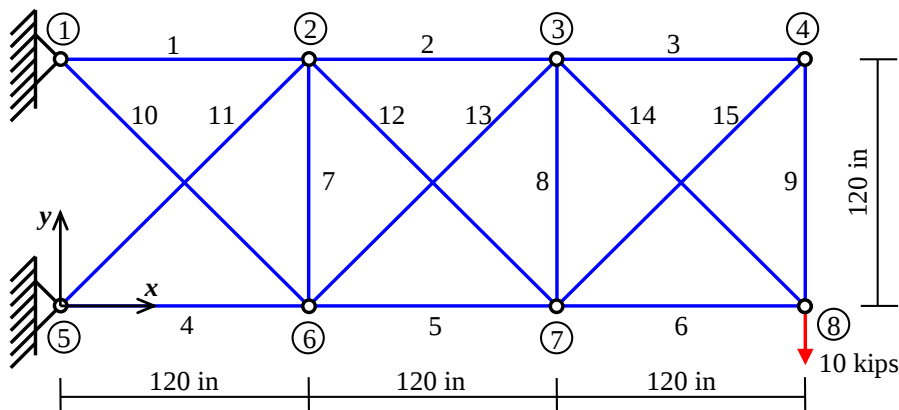


Figure 2. 15-bar planar truss structure

There are 23 design variables in this benchmark optimization including 15 sizing variables ( $A_i$ ,  $i=1,2,\dots,15$ ) and 8 configuration variables ( $x_2 = x_6$ ;  $x_3 = x_7$ ;  $y_2$ ;  $y_3$ ;  $y_4$ ;  $y_6$ ;  $y_7$ ;  $y_8$ ). The allowable stress for all elements is  $\pm 25$  ksi. Sizing variables are selected from the following discrete set during the optimization process:

$D = \{ 0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180 \}$  (in.<sup>2</sup>). In addition, side constraints for configuration variables are as follows:

100 in.  $\leq x_2 \leq 140$  in.; 220 in.  $\leq x_3 \leq 260$  in.; 100 in.  $\leq y_2 \leq 140$  in.; 100 in.  $\leq y_3 \leq 140$  in.; 50 in.  $\leq y_4 \leq 90$  in.; -20 in.  $\leq y_6 \leq 20$  in.; -20 in.  $\leq y_7 \leq 20$  in.; 20 in.  $\leq y_8 \leq 60$  in.;

In the literature, this benchmark truss optimization problem has been dealt with genetic algorithm (GA) [13], Firefly algorithm (FA) [14], modified harmony search algorithm (MHSA) [15], and artificial bee colony algorithm (ABCA) [3].

In this example, 50 independent optimization runs are conducted using CMO for each value of  $\alpha$ , and the best results are obtained when  $\alpha = 6.75$ . The number of particles and the maximum number of iterations are 50 and 200, respectively. The best results obtained by CMO are compared with those of other algorithms in Table 1. In addition, the best layout found by the CMO is shown in Fig. 3.

Table 1: Optimal designs of 15-bar planar truss

| No.              | Design Variable | GA [13]  | FA [14] | MHSA [15] | ABCA [3] | CMO      |
|------------------|-----------------|----------|---------|-----------|----------|----------|
| 1                | $A_1$           | 1.081    | 0.954   | 0.954     | 0.954    | 0.954    |
| 2                | $A_2$           | 0.539    | 0.539   | 0.539     | 0.539    | 0.539    |
| 3                | $A_3$           | 0.287    | 0.220   | 0.220     | 0.347    | 0.174    |
| 4                | $A_4$           | 0.954    | 0.954   | 0.954     | 0.954    | 0.954    |
| 5                | $A_5$           | 0.539    | 0.539   | 0.539     | 0.539    | 0.539    |
| 6                | $A_6$           | 0.141    | 0.220   | 0.220     | 0.111    | 0.270    |
| 7                | $A_7$           | 0.111    | 0.111   | 0.111     | 0.111    | 0.111    |
| 8                | $A_8$           | 0.111    | 0.111   | 0.111     | 0.111    | 0.111    |
| 9                | $A_9$           | 0.539    | 0.287   | 0.440     | 0.539    | 0.174    |
| 10               | $A_{10}$        | 0.440    | 0.440   | 0.347     | 0.440    | 0.347    |
| 11               | $A_{11}$        | 0.539    | 0.440   | 0.347     | 0.440    | 0.347    |
| 12               | $A_{12}$        | 0.270    | 0.220   | 0.270     | 0.174    | 0.220    |
| 13               | $A_{13}$        | 0.220    | 0.220   | 0.270     | 0.174    | 0.220    |
| 14               | $A_{14}$        | 0.141    | 0.270   | 0.220     | 0.111    | 0.270    |
| 15               | $A_{15}$        | 0.287    | 0.220   | 0.220     | 0.347    | 0.174    |
| 16               | $x_2$           | 101.5775 | 114.967 | 135.5676  | 110.2086 | 135.2451 |
| 17               | $x_3$           | 227.9112 | 247.040 | 245.5421  | 249.8193 | 257.1279 |
| 18               | $y_2$           | 134.7986 | 125.919 | 123.1303  | 133.5991 | 120.3808 |
| 19               | $y_3$           | 128.2206 | 111.067 | 120.6957  | 111.6235 | 107.8293 |
| 20               | $y_4$           | 54.86300 | 58.298  | 57.9313   | 55.1278  | 50.8612  |
| 21               | $y_6$           | -16.4484 | -17.564 | -5.9742   | -18.9505 | -9.5547  |
| 22               | $y_7$           | -13.3007 | -5.821  | -2.9125   | 3.3411   | -0.1707  |
| 23               | $y_8$           | 54.8572  | 31.465  | 56.3256   | 55.1423  | 50.8125  |
| Best weight (lb) |                 | 76.68    | 75.55   | 73.887    | 72.715   | 72.267   |
| Analyses         |                 | 8,000    | 8,000   | 5,000     | 18,000   | 10,000   |

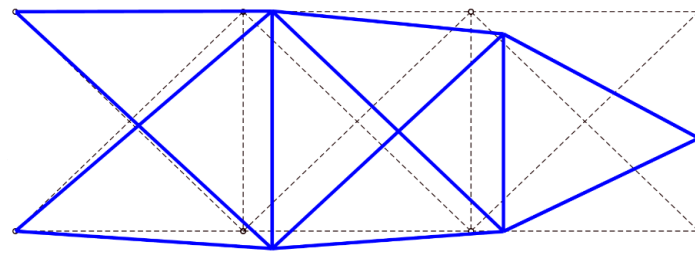


Figure 3. Optimum layout of 15-bar planar truss found by CMO

Based on the results given in Table 1, it can be concluded that the CMO algorithm is more effective than all other algorithms in finding the best solution. Additionally, the 50 independent optimization runs produced the best weight of 72.267 kg, the worst weight of 83.937 kg, the average weight of 79.007 kg, and a corresponding standard deviation of 2.006 kg. Furthermore, Fig. 4 presents the optimum weights obtained by the CMO algorithm in 50 independent optimization runs.

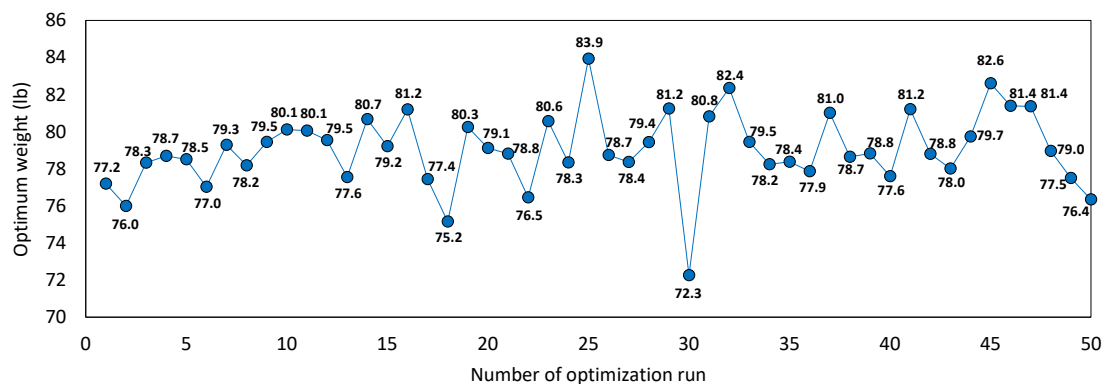


Figure 4. Optimum weights obtained by CMO for 15-bar planar truss

Fig. 5 presents the convergence curves of all optimization runs, along with the best and mean convergence curves. It can be seen that the mean convergence curve is very close to the best convergence curve indicating the good performance of the CMO in all the independent optimization runs.

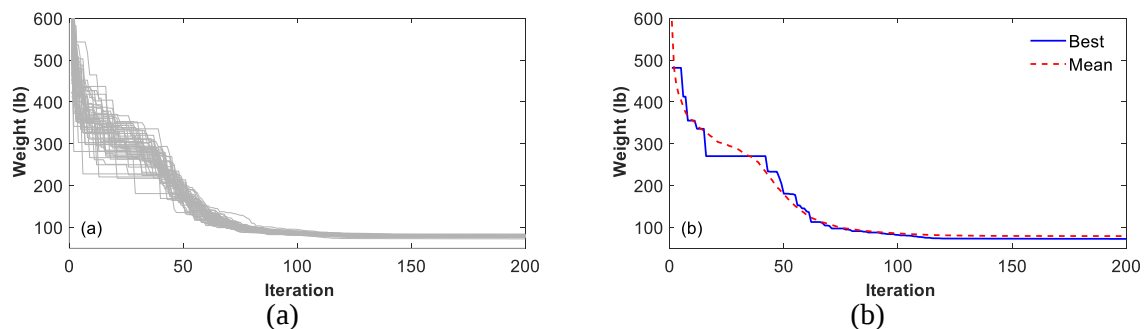


Figure 5. (a) All convergence curves and (b) the best and mean convergence curves of CMO in layout optimization of 15-bar planar truss

#### 4.2 18-bar truss

The second design problem is a planar 18-bar truss shown in Fig. 6. The material density and the modulus of elasticity are  $0.1 \text{ lb/in}^3$  is  $10^4 \text{ ksi}$ , respectively.

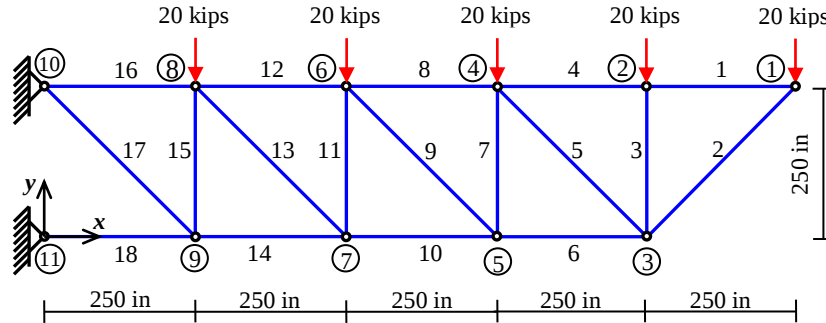


Figure 6. 18-bar planar truss structure

The members of this truss structure are divided into four design groups including (1)  $A_1 = A_4 = A_8 = A_{12} = A_{16}$ ; (2)  $A_2 = A_6 = A_{10} = A_{14} = A_{18}$ ; (3)  $A_3 = A_7 = A_{11} = A_{15}$ ; and (4)  $A_5 = A_9 = A_{13} = A_{17}$ . Additionally, there are eight layout design variables including  $x_3; y_3; x_5; y_5; x_7; y_7; x_9; y_9$ . As a result, the layout optimization problem of the 18-bar planar truss has 12 design variables. The allowable stresses for all elements are  $\pm 25 \text{ ksi}$  and  $4EA/L^2$  (Euler buckling stress). Sizing design variables are selected from the following discrete set during the optimization process:  $D = \{2.00, 2.25, \dots, 21.50, 21.75\} \text{ (in.}^2\text{)}$ . In addition, side constraints for configuration variables are as follows:  $775 \text{ in.} \leq x_3 \leq 1225 \text{ in.}$ ;  $525 \text{ in.} \leq x_5 \leq 975 \text{ in.}$ ;  $275 \text{ in.} \leq x_7 \leq 725 \text{ in.}$ ;  $25 \text{ in.} \leq x_9 \leq 475 \text{ in.}$ ;  $-225 \text{ in.} \leq y_3, y_5, y_7, y_9 \leq 245 \text{ in.}$

In the literature, researchers have solved this benchmark truss layout optimization problem using simulated annealing (SA) [16], GA [13], group search optimization (GSO) [17], and ABCA [3].

Table 2: Optimal designs of 18-bar planar truss

| No.              | Design Variable | SA [16] | GA [13]  | GSO [17]  | ABCA [3] | CMO      |
|------------------|-----------------|---------|----------|-----------|----------|----------|
| 1                | $A_1$           | 12.25   | 12.75    | 12.25     | 12.50    | 12.00    |
| 2                | $A_2$           | 17.50   | 18.50    | 18.25     | 17.75    | 18.00    |
| 3                | $A_3$           | 5.75    | 4.75     | 4.75      | 5.75     | 5.00     |
| 4                | $A_5$           | 4.25    | 3.25     | 4.25      | 3.75     | 4.50     |
| 5                | $x_3$           | 910.0   | 917.4475 | 916.9     | 912.9974 | 915.0135 |
| 6                | $y_3$           | 179.0   | 193.7899 | 191.971   | 183.6806 | 188.9937 |
| 7                | $x_5$           | 638.0   | 654.3243 | 654.224   | 642.7143 | 648.7662 |
| 8                | $y_5$           | 141.0   | 159.9436 | 156.1     | 143.8920 | 150.9643 |
| 9                | $x_7$           | 408.0   | 424.4821 | 423.5     | 411.6918 | 418.3478 |
| 10               | $y_7$           | 91.0    | 108.5779 | 102.571   | 97.14763 | 97.3077  |
| 11               | $x_9$           | 198.0   | 208.4691 | 207.519   | 200.9087 | 205.5284 |
| 12               | $y_9$           | 24.0    | 37.6349  | 28.579    | 30.21906 | 23.0166  |
| Best weight (lb) |                 | 4533.24 | 4530.68  | 4538.7676 | 4537.064 | 4525.864 |
| Analyses         |                 | -       | 8,000    | -         | 18,000   | 10,000   |

The CMO is used to conduct 50 independent optimization runs for each value of  $\alpha$ . It is observed that the best results are obtained when  $\alpha$  is set to 6.75. For the layout optimization, 50 particles and 200 iterations are considered. Table 2 compares the best results obtained by CMO with other algorithms. Additionally, Fig. 7 shows the best layout found by the CMO.

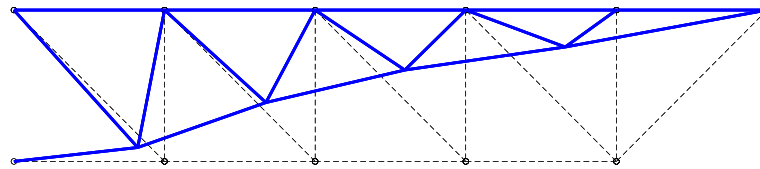


Figure 7. Optimum layout of 18-bar planar truss found by CMO

The optimization results indicate that the CMO algorithm outperforms all other algorithms in finding the best solution. Fig. 8 displays optimum weights obtained by the CMO algorithm in 50 independent optimization runs. The best, worst, average weights and their standard deviation are 4525.864, 4714.999, 4586.438, and 37.6838 kg, respectively.

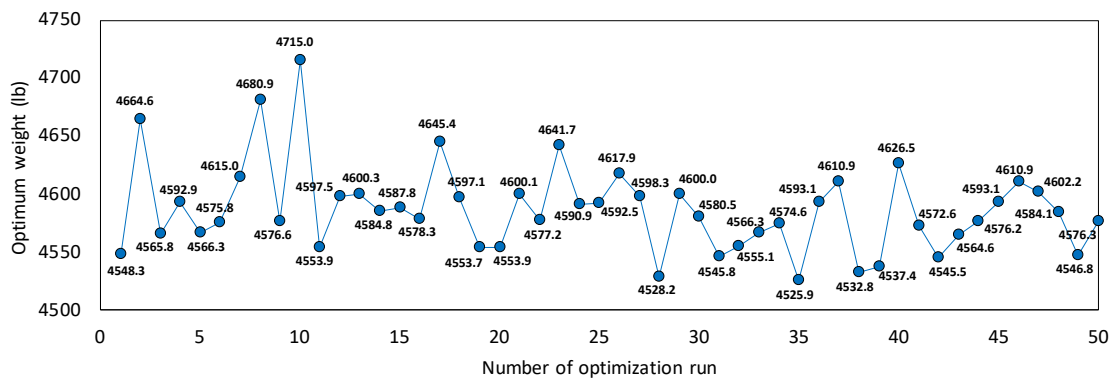


Figure 8. Optimum weights obtained by CMO for 18-bar planar truss

In this example, the convergence curves of all optimization runs, along with the best and mean convergence curves are displayed in Fig. 9. It can be observed that the CMO performed well in all independent optimization runs, as the mean convergence curve is very close to the best convergence curve.

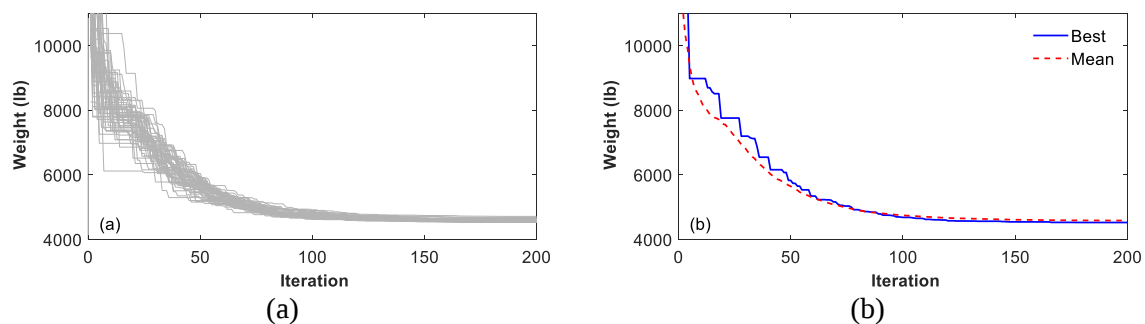


Figure 9. (a) All convergence curves and (b) the best and mean convergence curves of CMO in layout optimization of 18-bar planar truss

### 4.3 47-bar truss

The third design problem of this paper involves a planar 47-bar tower shown in Fig. 10. The material density and the modulus of elasticity are  $0.3 \text{ lb/in}^3$  and  $3 \times 10^5 \text{ ksi}$ , respectively.

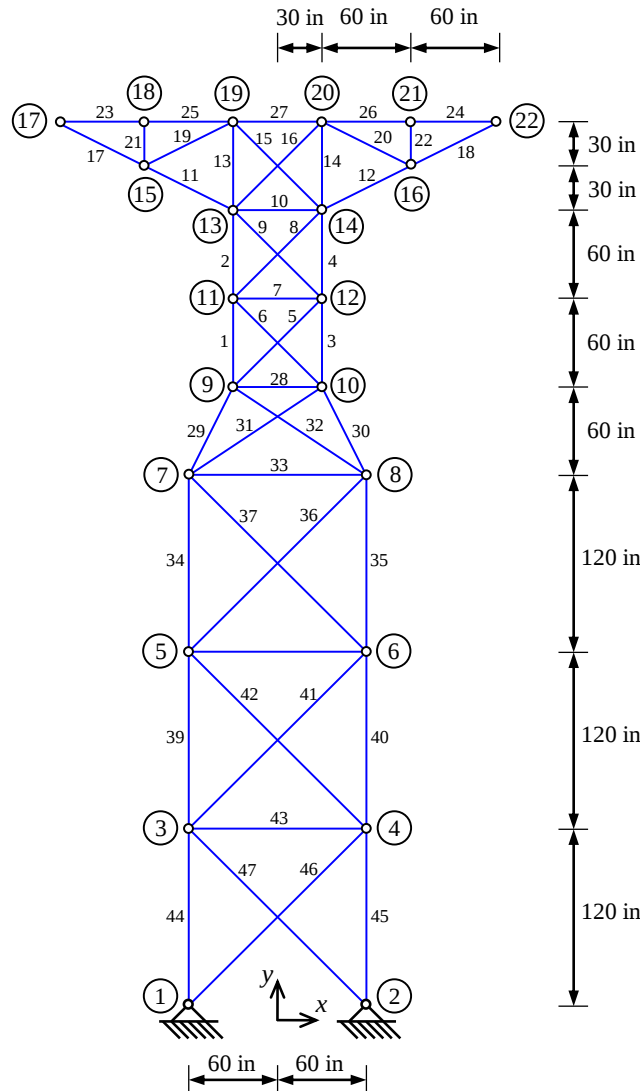


Figure 10. 47-bar planar truss structure

Three different loading conditions are applied independently to the tower. The first condition is a concentrated load of 6000 lbf in the positive  $x$ -direction and another 14,000 lbf in the negative  $y$ -direction applied to nodes 17 and 22. The second condition is a concentrated load of 6000 lbf in the positive  $x$ -direction and 14,000 lbf in the negative  $y$ -direction applied to node 17. The third condition is a concentrated load of 6000 lbf in the positive  $x$ -direction and a concentrated force of 14,000 lbf in the negative  $y$ -direction applied to node 22.

Table 3: Optimal designs of 47-bar planar truss

| No.             | Design Variable | BB [18] | GA [1]  | SA [16] | ABCA [3] | CMO      |
|-----------------|-----------------|---------|---------|---------|----------|----------|
| 1               | $A_1$           | 2.61    | 2.50    | 2.50    | 2.4      | 2.8      |
| 2               | $A_2$           | 2.56    | 2.20    | 2.50    | 2.2      | 2.6      |
| 3               | $A_5$           | 0.69    | 0.70    | 0.80    | 1.1      | 0.6      |
| 4               | $A_7$           | 0.47    | 0.10    | 0.10    | 0.1      | 0.1      |
| 5               | $A_8$           | 0.80    | 1.30    | 0.70    | 1.2      | 0.8      |
| 6               | $A_{10}$        | 1.13    | 1.30    | 1.30    | 1.3      | 1.0      |
| 7               | $A_{12}$        | 1.71    | 1.80    | 1.80    | 1.7      | 1.6      |
| 8               | $A_{14}$        | 0.77    | 0.50    | 0.70    | 0.6      | 0.9      |
| 9               | $A_{15}$        | 1.09    | 0.80    | 0.90    | 0.8      | 1.0      |
| 10              | $A_{18}$        | 1.34    | 1.20    | 1.20    | 1.6      | 1.2      |
| 11              | $A_{20}$        | 0.36    | 0.40    | 0.40    | 0.3      | 0.3      |
| 12              | $A_{22}$        | 0.97    | 1.20    | 1.30    | 0.9      | 1.0      |
| 13              | $A_{24}$        | 1.00    | 0.90    | 0.90    | 1.2      | 0.9      |
| 14              | $A_{26}$        | 1.03    | 1.00    | 0.90    | 1.0      | 0.9      |
| 15              | $A_{27}$        | 0.88    | 3.60    | 0.70    | 1.0      | 0.9      |
| 16              | $A_{28}$        | 0.55    | 0.10    | 0.10    | 0.6      | 0.1      |
| 17              | $A_{30}$        | 2.59    | 2.40    | 2.50    | 2.8      | 2.7      |
| 18              | $A_{31}$        | 0.84    | 1.10    | 1.00    | 0.4      | 1.0      |
| 19              | $A_{33}$        | 0.25    | 0.10    | 0.10    | 0.1      | 0.1      |
| 20              | $A_{35}$        | 2.86    | 2.70    | 2.90    | 2.9      | 2.9      |
| 21              | $A_{36}$        | 0.92    | 0.80    | 0.80    | 1.5      | 1.1      |
| 22              | $A_{38}$        | 0.67    | 0.10    | 0.10    | 0.6      | 0.1      |
| 23              | $A_{40}$        | 3.06    | 2.80    | 3.00    | 3.1      | 3.1      |
| 24              | $A_{41}$        | 1.04    | 1.30    | 1.20    | 0.9      | 0.9      |
| 25              | $A_{43}$        | 0.10    | 0.20    | 0.10    | 0.1      | 0.1      |
| 26              | $A_{45}$        | 3.13    | 3.00    | 3.20    | 3.3      | 3.2      |
| 27              | $A_{46}$        | 1.12    | 1.20    | 1.10    | 0.8      | 1.2      |
| 28              | $x_2$           | 107.76  | 114.0   | 104.0   | 103.6063 | 102.4999 |
| 29              | $x_4$           | 89.15   | 97.0    | 87.0    | 81.5008  | 84.0721  |
| 30              | $y_4$           | 137.98  | 125.0   | 128.0   | 143.0525 | 141.7668 |
| 31              | $x_6$           | 66.75   | 76.0    | 70.0    | 67.0169  | 70.8394  |
| 32              | $y_6$           | 254.47  | 261.0   | 259.0   | 252.8466 | 234.1873 |
| 33              | $x_8$           | 57.38   | 69.0    | 62.0    | 54.5203  | 54.3291  |
| 34              | $y_8$           | 342.16  | 316.0   | 326.0   | 374.0126 | 347.9126 |
| 35              | $x_{10}$        | 49.85   | 56.0    | 53.0    | 39.8226  | 44.8092  |
| 36              | $y_{10}$        | 417.17  | 414.0   | 412.0   | 443.9461 | 441.7606 |
| 37              | $x_{12}$        | 44.66   | 50.0    | 47.0    | 30.9474  | 41.0135  |
| 38              | $y_{12}$        | 475.35  | 463.0   | 486.0   | 491.9941 | 487.9129 |
| 39              | $x_{14}$        | 41.09   | 54.0    | 45.0    | 36.7597  | 41.6433  |
| 40              | $y_{14}$        | 513.15  | 524.0   | 504.0   | 510.000  | 518.0844 |
| 41              | $x_{20}$        | 17.90   | 1.0     | 2.0     | 17.6763  | 1.0137   |
| 42              | $y_{20}$        | 597.92  | 587.0   | 584.0   | 598.8911 | 603.1467 |
| 43              | $x_{21}$        | 93.54   | 99.0    | 89.0    | 77.6661  | 98.0282  |
| 44              | $y_{21}$        | 623.94  | 631.0   | 637.0   | 619.8911 | 631.3745 |
| Best weight(lb) |                 | 1900.00 | 1925.79 | 1871.70 | 1871.843 | 1871.96  |
| Analyses        |                 | -       | 100,000 | -       | 18,000   | 15,000   |

The structural elements are divided into 27 sizing design variables as follows:  $A_3 = A_1$ ;  $A_4 = A_2$ ;  $A_5 = A_6$ ;  $A_7, A_8 = A_9$ ;  $A_{10}, A_{12} = A_{11}$ ;  $A_{14} = A_{13}$ ;  $A_{15} = A_{16}$ ;  $A_{18} = A_{17}$ ;  $A_{20} = A_{19}$ ;  $A_{22} = A_{21}$ ;  $A_{24} = A_{23}$ ;  $A_{26} = A_{25}$ ;  $A_{27}, A_{28}$ ;  $A_{30} = A_{29}$ ;  $A_{31} = A_{32}$ ;  $A_{33}, A_{35} = A_{34}$ ;  $A_{36} = A_{37}$ ;  $A_{38}, A_{40} = A_{39}$ ;  $A_{41} = A_{42}$ ;  $A_{43}, A_{45} = A_{44}$ ;  $A_{46} = A_{47}$ ; Sizing design variables are selected from the following discrete set:  $D = \{0.1, 0.2, 0.3, \dots, 4.8, 4.9, 5.0\}$  (in.<sup>2</sup>). Nodes 15, 16, 17 and 22 are fixed while only  $x$ -coordinates of nodes 1 and 2 can be changed. Therefore, 17 layout design variables are:  $x_2 = -x_1$ ;  $x_4 = -x_3$ ;  $y_4 = y_3$ ;  $x_6 = -x_5$ ;  $y_6 = y_5$ ;  $x_8 = -x_7$ ;  $y_8 = y_7$ ;  $x_{10} = -x_9$ ;  $y_{10} = y_9$ ;  $x_{12} = -x_{11}$ ;  $y_{12} = y_{11}$ ;  $x_{14} = -x_{13}$ ;  $y_{14} = y_{13}$ ;  $x_{20} = -x_{19}$ ;  $y_{20} = y_{19}$ ;  $x_{21} = -x_{18}$ ;  $y_{21} = y_{18}$ ;  $x_i, y_i \in R$ . The tension and compression allowable stresses of members are 20 and 15 ksi, respectively. In addition, Euler buckling stress for each member is  $3.96EA/L^2$ .

In the literature, researchers have solved this benchmark truss layout optimization problem using the branch and bound (BB) method [18], GA [1], SA [16], and ABCA [3].

The CMO is utilized to perform 50 independent optimization runs for each value of  $\alpha$ , and  $\alpha=6.75$  gives the best results, similar to the other examples. In this example, 50 particles and 300 iterations are used. Table 3 presents the best solution for CMO and other algorithms in the literature. Additionally, the best layout found for the tower is displayed in Fig. 11.

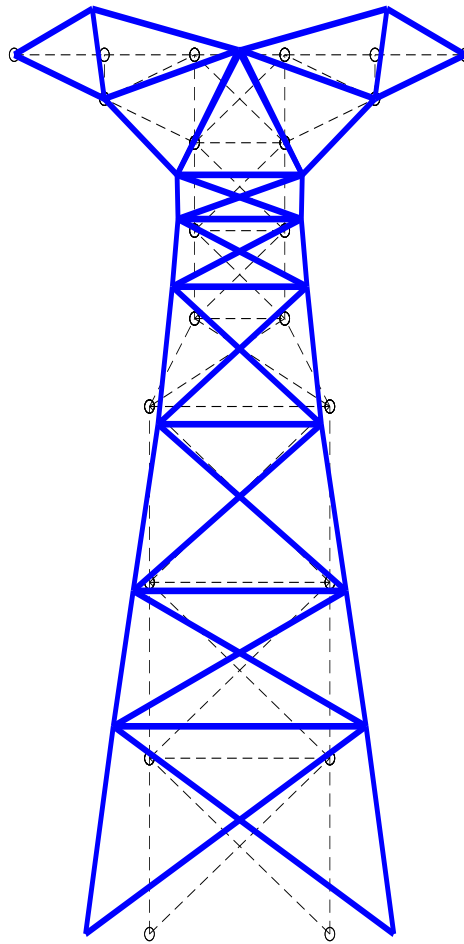


Figure 11. Optimum layout of 47-bar tower found by CMO

The optimization results indicate that the CMO algorithm outperforms GA [1] and BB [18] and is competitive with ABCA [3] and SA [16] in finding the best solution. The weights of all the optimal solutions found are presented in Fig. 12. The best, worst, average weights and their standard deviation are 1871.964, 1996.998, 1934.032, and 22.451 kg, respectively.

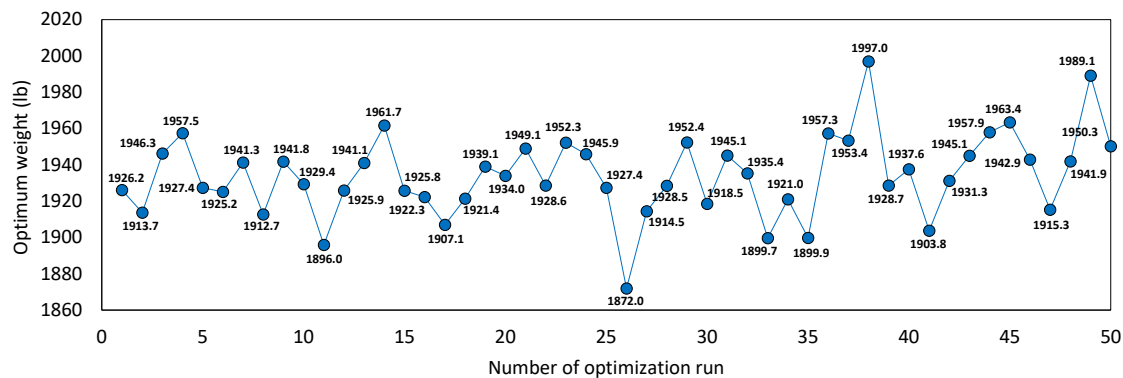


Figure 12. Optimum weights obtained by CMO for 47-bar tower

Fig. 13 shows the convergence curves of all optimization runs, along with the best and mean convergence curves are displayed. Obviously, the CMO performed well in all the optimization runs, because the mean convergence curve is very close to the best convergence curve.

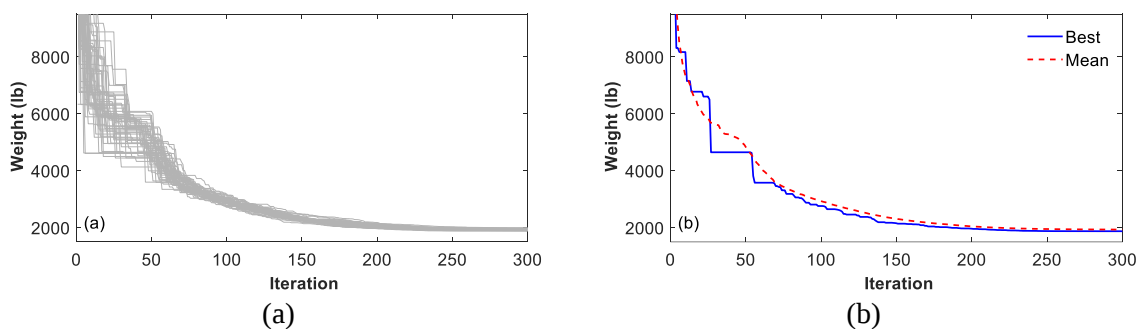


Figure 13. (a) All convergence curves and (b) the best and mean convergence curves of CMO in layout optimization of 47-bar tower

## 5. CONCLUSIONS

This paper presented a center of mass optimization (CMO) metaheuristic algorithm for sizing-layout optimization of planar truss structures. The CMO metaheuristic is based on the physical concept of the center of mass in space. Recent studies have shown that CMO is highly effective in solving various types of structural optimization problems compared to other metaheuristics. This paper applies the CMO to deal with the layout optimization problem of planar truss structures. The optimization process takes into account both discrete

and continuous design variables. The cross-sectional areas of the truss are considered discrete design variables, while the coordinates of structural joints are treated as continuous design variables.

This paper presents three benchmark design examples of truss layout optimization including 15, 18, and 47-bar trusses. The optimization results for the truss layout obtained by the CMO algorithm are compared with those obtained by other optimization techniques in the literature. The numerical analysis shows that for 15-bar and 18-bar trusses, the CMO outperforms the artificial bee colony algorithm (ABCA) [3], genetic algorithm (GA) [13], firefly algorithm (FA) [14], modified harmony search algorithm (MHSA) [15], simulated annealing (SA) [16], and group search optimization (GSO) [17]. On the other hand, for the 47-bar truss, the CMO outperforms a genetic algorithm (GA) [1] and branch and bound (BB) [18] method, and is competitive with ABCA and SA in finding the best solution. These findings indicate that the CMO is an effective optimization algorithm for truss layout optimization.

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